Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

Institut für Integrierte Systeme
Integrated Systems Laboratory

Simulation of Noise in Semiconductor Devices with Dessis$_{ISE}$ Using the Direct Impedance Field Method

Bernhard Schmithüsen, Andreas Schenk, Wolfgang Fichtner

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Abstract

A detailed description of the Dessis$_{ISE}$ implementation of a direct impedance field method for noise simulation in physical semiconductor devices is presented. The Green function approach to the Langevin equation of the phenomenological system equations is described and applied to the semiconductor device equations. Some physical noise source models in devices are summarized and the noise figure of two-port devices is recapitulated. The numerical algorithm to extract the Green functions is described. The last part may serve as a manual for noise simulations with Dessis$_{ISE}$.

Contents

1 Introduction

The characterization of device behavior through rigorous simulation of the underlying phenomenological partial differential equations (PDEs) became maturated in the last years. Based on the van Roosbroeck's drift diffusion model or more sophisticated thermodynamic resp. hydrodynamic models the state-of-the-art simulation includes advanced physical phenomena. Nevertheless the statistical behavior of the carriers is seldomly taken into account - so higher order effects and perturbations of the idealized solutions are ignored though they determine to a certain amount the reliability and functionality of modern semiconductor devices.
In recent years some effort has been devoted to the numerical simulation of noise phenomena in physics based device simulators. In most cases the noise simulation is founded on Shockley's impedance field method [1] and its variations and generalizations. Bonani, Ghione, Pinto and Smith [2] reported a numerically efficient Green function approach to the Langevin equation based simulation of the impedance field method (IFM) which is the basis for the implementation in the multi-dimensional, mixed-mode device simulator Dessis$_{ISE}$ described here, and is a variation of the *direct impedance field method* (DIFM). The so called *adjoint impedance field method* (AIFM) (e.g. [3]) is limited to one-carrier devices and had been earlier implemented into Dessis$_{ISE}$ [4].

2 Direct Impedance Field Method

The noise analysis is based on the Langevin equation using a Green function approach. It has been shown that the Green function approach is equivalent to Shockley's direct impedance field method (see [5]). This technique allows the modeling of small-signal perturbations of the underlying transport model and to compute the voltage and current fluctuations at the terminals in terms of correlation spectra due to the local microscopic noise sources in the device.

2.1 The Langevin Approach

The physical system of interest is described by a system of time-dependent, nonlinear PDEs which can abstractly be written in the form

$$F(D, u) = 0 \qquad . \tag{1}$$

These phenomenological equations are completed by suitable boundary conditions. Let $u_0 = u_0(t, x)$ be a solution.
In the Langevin approach these phenomenogical equations are perturbed by small excitations, random forces or "Langevin forces" s and the Langevin equation takes the form

$$F(D, u_0 + \delta u) = s$$

For consistency with the PDE the random forces have zero mean value, i.e. $\langle s_t \rangle = 0$ for all times t (here $\langle \cdot \rangle$ indicates the integration over the underlying event space). Furthermore the Langevin forces are not (time) correlated, i.e. $\langle s_t s_{t'} \rangle = A\delta(t - t')$, so the Langevin sources are *white*. Within the formalism of the master equation a relation between A and the second Fokker-Planck moment can be established ([6]), i.e. the spectra of the random forces are in principle known.

Under the assumption of small perturbations the system can be linearized taking the form

$$L(D, u_0)\delta u = s \qquad . \tag{2}$$

We consider the Green functions G of the linearized equation, i.e. the solution of

$$L(D, u_0)G(x, x_1; t, t_1) = \delta(t - t_1)\delta(x - x_1) \tag{3}$$

such that the solution δu can formally be written as

$$\delta u(x,t) = \int_\Omega \int_{-\infty}^{t} G(x, x_1; t, t_1)s(x_1, t_1)dt_1 dx_1 \qquad .$$

For a stationary solution u_0 the linearized equation (2) becomes time-invariant and for the Green functions one obtains $G(x, x_1; t, t_1) = G(x, x_1; t - t_1, 0)$, so a frequency domain analysis becomes possible, and Fourier transformation implies ($\widehat{f}$ denotes the Fourier transformation of f)

$$\widehat{\delta u}(x, \omega) = \int_\Omega \widehat{G}(x, x_1; \omega)\widehat{s}(x_1, \omega)dx_1$$

and the correlation spectrum can be recovered as

$$S_{\delta u, \delta u}(x, x'; \omega) = \int_\Omega \int_\Omega \widehat{G}(x, x_1; \omega)S_{s,s}(x_1, x_2; \omega)\widehat{G}^*(x', x_2; \omega)dx_1 dx_2 \qquad , \tag{4}$$

where $S_{s,s}$ denotes the correlation spectrum of the Langevin forces.

2.2 Langevin Approach for the Device Equations

For the further discussion in this section we use the basic van Roosbroeck's drift diffusion model. Straight forward extensions of the choosen approach to more extended transport models are obvious. The model equations can be formulated as

$$\begin{aligned} -\nabla(\epsilon\nabla\psi) &= q(p - n + N) \\ q\frac{\partial n}{\partial t} - \nabla J_n &= -qR \\ q\frac{\partial p}{\partial t} + \nabla J_p &= -qR \end{aligned} \tag{5}$$

with the usual meaning of the symbols. Concerning the linearized system we are solving the Langevin equations

$$\begin{aligned} L_\psi(D, u_0)\delta u &= s_\psi \\ L_n(D, u_0)\delta u &= s_n \\ L_p(D, u_0)\delta u &= s_p \end{aligned} \tag{6}$$

where $\delta u = (\delta\psi, \delta n, \delta p)$, resulting in the correlation spectra $(\alpha = \psi, n, p)$

$$S_{\delta\alpha, \delta\beta}(x, x'; \omega) = \sum_{\gamma, \delta = \psi, n, p} \int_\Omega \int_\Omega \widehat{G^\alpha_\gamma}(x, x_1; \omega)S_{s_\gamma, s_\delta}(x_1, x_2; \omega)\widehat{G^\beta_\delta}^*(x', x_2; \omega)dx_1 dx_2 \qquad . \tag{7}$$

The nature of our equations suggests to split the noise source term for the continuity equations into parts reflecting the generation-recombination and the current density fluctuations

$$s_\alpha = \gamma_\alpha + \nabla \xi_\alpha \quad (\alpha = n, p) \tag{8}$$

while for the Poisson equation $s_\alpha = 0$ is used (for extended transport models $s_\alpha = 0 \quad (\alpha \neq n, p)$ is assumed).
We define the vector Green function as

$$\underline{\widehat{G^{\alpha}_{\beta}}}(x, x_1; \omega) = \nabla_{x_1} \widehat{G^{\alpha}_{\beta}}(x, x_1; \omega) \quad (\beta = n, p) \quad . \tag{9}$$

Looking at the voltage correlation spectra at different locations x and x' we obtain under the assumption of independent noise sources γ and ξ

$$\begin{aligned} S_{\delta\psi,\delta\psi}(x, x'; \omega) &= \sum_{\alpha,\beta=n,p} \int_\Omega \int_\Omega \widehat{G^{\psi}_{\alpha}}(x, x_1; \omega) S_{s_\alpha, s_\beta}(x_1, x_2; \omega) \widehat{G^{\psi}_{\beta}}^*(x', x_2; \omega) dx_1 dx_2 \\ &= \sum_{\alpha,\beta=n,p} \int_\Omega \int_\Omega \widehat{G^{\psi}_{\alpha}}(x, x_1; \omega) S_{\gamma_\alpha, \gamma_\beta}(x_1, x_2; \omega) \widehat{G^{\psi}_{\beta}}^*(x', x_2; \omega) dx_1 dx_2 \\ &\quad + \sum_{\alpha,\beta=n,p} \int_\Omega \int_\Omega \underline{\widehat{G^{\psi}_{\alpha}}}(x, x_1; \omega) S_{\xi_\alpha, \xi_\beta}(x_1, x_2; \omega) \underline{\widehat{G^{\psi}_{\beta}}}^*(x', x_2; \omega) dx_1 dx_2 \quad . \end{aligned} \tag{10}$$

It has been shown that the Green function approach for the Langevin equation is equivalent to Shockley's Impedance Field Method [5], i.e. with $\kappa_n = -1$ and $\kappa_p = +1$ we have for the electron and hole impedance field $\widehat{Z_\alpha}$

$$\widehat{Z_\alpha}(x, x_1; \omega) = \kappa_\alpha \widehat{G^{\psi}_{\alpha}}(x, x_1; \omega) \quad (\alpha = n, p) \quad . \tag{11}$$

For not too small devices one can assume the sources to be spatially independent, i.e.

$$\begin{aligned} S_{\gamma_\alpha, \gamma_\beta}(x_1, x_2; \omega) &= K_{\gamma_\alpha, \gamma_\beta}(x_1; \omega) \cdot \delta(x_1 - x_2) \\ \underline{\underline{S}}_{\xi_\alpha, \xi_\beta}(x_1, x_2; \omega) &= \underline{\underline{K}}_{\xi_\alpha, \xi_\beta}(x_1; \omega) \cdot \delta(x_1 - x_2) \end{aligned} \tag{12}$$

where K and $\underline{\underline{K}}$ are called the local GR noise source K and the local current density noise source $\underline{\underline{K}}$, respectively.
This finally ends up in the formula

$$\begin{aligned} S_{\delta\psi,\delta\psi}(x, x'; \omega) &= \sum_{\alpha,\beta=n,p} \int_\Omega \widehat{G^{\psi}_{\alpha}}(x, x_1; \omega) K_{\gamma_\alpha, \gamma_\beta}(x_1; \omega) \widehat{G^{\psi}_{\beta}}^*(x', x_1; \omega) dx_1 \\ &\quad + \sum_{\alpha,\beta=n,p} \int_\Omega \underline{\widehat{G^{\psi}_{\alpha}}}(x, x_1; \omega) \underline{\underline{K}}_{\xi_\alpha, \xi_\beta}(x_1; \omega) \underline{\widehat{G^{\psi}_{\beta}}}^*(x', x_1; \omega) dx_1 \end{aligned} \tag{13}$$

which is the classical expression for noise within the impedance field method.

3 Noise Sources

The noise sources can take the form of scalar GR noise sources K or of tensor current density noise sources $\underline{\underline{K}}$. There units are given by

$$[K] = 1\frac{C^2}{m^3 s} \quad , \quad [\underline{\underline{K}}] = 1\frac{C^2}{ms} \tag{14}$$

The noise sources are *white* if they do not depend on frequency. Physical considerations lead to either noise models in the GR or current density noise source form.

3.1 Diffusion Noise

Diffusion noise is due to fluctuations of the velocities of the carriers, caused by collisions with phonons, impurities, etc. The following expression for the electron diffusion noise source can be derived (e.g. Nougier [3])

$$\underline{\underline{K}}^{diff} = 4q^2 n D_n \tag{15}$$

where n is the electron density, D_n the electron diffusivity, and q the elementary charge. One has to understand the right hand side of the equation as a diagonal tensor with diagonal entries equal to $4q^2 n D_n$ and zero offdiagonals. Observe that correlations between the carriers, i.e. carrier-carrier scattering, are neglected and in addition anisotropic effects are not taken into account.

3.2 Generation-Recombination (GR) Noise

Local fluctuations of the carrier densities give rise to so called GR noise sources. With respect to the different mechanisms of GR processes, e.g. SRH recombination, band-to-band recombination, avalanche generation etc., the noise source models have to be developed. Often the GR noise is expressed in local current density noise sources already partially containing the response of the device. Bonani and Ghione [7] discuss several GR noise processes in detail.

Equivalent Monopolar GR Noise Source

An equivalent monopolar GR noise source model (e.g. Bonani-Ghione [7], Nougier [3]) for a two-level generation-recombination process can be derived as

$$\underline{\underline{K}}^{GR}(x, f) = \frac{J_n J_n}{n} \frac{4\alpha\tau_{eq}}{1 + \omega^2 {\tau_{eq}}^2} \tag{16}$$

where J_n is the electron current density, n the electron density, α a fitting parameter, and τ_{eq} an equivalent GR lifetime. The noise source is an equivalent current density noise source for GR processes and already includes partially the response of the device, implying that it is frequency dependent and not white.

Bulk Flicker GR-Noise

Taking a range of GR lifetimes into account a flicker GR noise model can be derived (van der Ziel [8], p. 125 ff)

$$K^{fGR}(x,f) = \frac{J_{n_0} J_{n_0}}{n} \frac{2\alpha_H}{\pi f \ln(\tau_1/\tau_0)} \left(arctan(\omega\tau_1) - arctan(\omega\tau_0)\right) \tag{17}$$

where α_H is a parameter, $\omega = 2\pi f$, and the time constants fulfill $\tau_0 < \tau_1$. With increasing frequency the noise source changes from constant to $1/f$ behavior close at the frequency $f_1 = 1/\tau_1$, and finally at frequency $f_0 = 1/\tau_0$ to a $1/f^2$ range.

4 Noise Figure

The *noise figure* is a figure of merit for the high frequency noise of two-port devices. Fig. 1 shows an equivalent network representing the principal noise measurement circuitry neglecting the dc biasing circuit and other parasitic parts. The spectrum of the noise power $P = v_O i_O^*$ at the output N_O of the device can be measured and the noise figure NF is then defined as

$$NF = \frac{P^{O,noisy}}{P^{O,noiseless}} \quad . \tag{18}$$

It depends on the frequency and the admittance of the input noise source Y_S. Here $P^{O,noisy}$ is the output noise power if the DUT is noisy while $P^{O,noiseless}$ is the output noise power for noiseless DUT. The standarized value for the input noise impedance $Z_S = 1/Y_S$ is 50 Ohm (pure real) resulting in the noise figure $NF50$. Optimizing this input impedance gives the *minimum noise figure* $NFmin$.
In the noise measurement circuit of Fig. 1 the noisy two-port is represented by its Y-parameters and equivalent voltage noise sources e_1 and e_2 at the input and output of the device. These voltage noise sources e_1 and e_2 with the noise voltage auto-correlation spectra S_V^1 and S_V^2 are correlated through the noise voltage cross-correlation spectrum S_V^{21}. Alternatively, the voltage noise sources e_1 and e_2 could

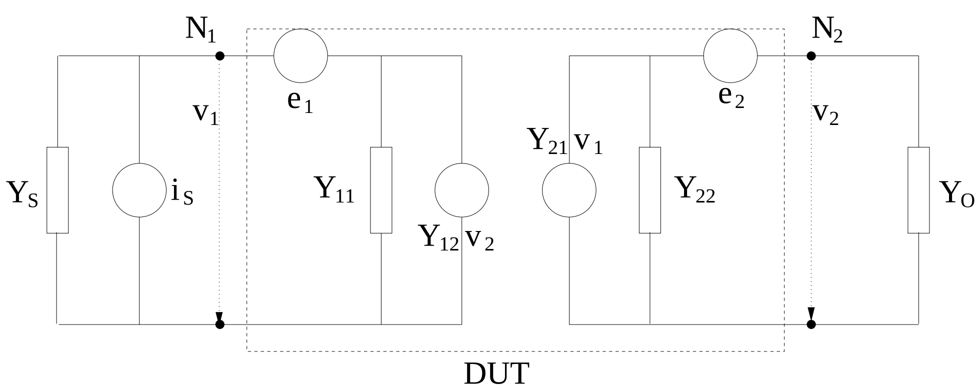

Figure 1: Equivalent network for principal noise measurement circuitry for a noisy two-port device (DUT) with voltage noise sources e_1, e_2, and input current noise source i_S.

be replaced by current noise sources $i_1 = Y_{11} e_1$ and $i_2 = Y_{22} e_2$ inserted in parallel to the admittances

Y_{11} and Y_{22}, respectively. The auto-correlation spectra of these current noise sources are $S_I^1 = |Y_{11}|^2 S_V^1$ and $S_I^2 = |Y_{22}|^2 S_V^2$, respectively, and their cross-correlation is given by $S_I^{21} = Y_{22} Y_{11}^* S_V^{21}$.
The noisy input admittance Y_S has a current noise spectrum $S_I^S = 4k_B T Re(Y_S)$ and is therefore complemented by the input current noise source i_S.
The noise figure NF can be derived as

$$NF = 1 + \frac{1}{S_I^S} \left(S_I^1 + \left| \frac{Y_s + Y_{11}}{Y_{21}} \right|^2 S_I^2 - 2Re\left(\frac{Y_s + Y_{11}}{Y_{21}} S_I^{21} \right) \right) \quad . \tag{19}$$

The noise figure NF has exactly one mimimum for positive real part $Re(Y_S)$ of the input admittance. Minimizing the noise figure NF wrt the input admittance Y_S gives the optimal input admittance Y_{min} as

$$Re(Y_{min}) = \frac{1}{S_I^d} \sqrt{\left(Re(Y_{11}) S_I^d - Re(S_I^{dg} Y_{21}^*) \right)^2 + |Y_{21}|^2 \left(S_I^d S_I^g - |S_I^{dg}|^2 \right)} \tag{20}$$

$$Im(Y_{min}) = -Im(Y_{11}) - \frac{1}{S_I^d} Im(S_I^{dg} Y_{21}^*) \quad . \tag{21}$$

5 Numerical Approach

For the numerical computation of the device Green functions for each observation node an efficient algorithm based on a block decomposition of the (Fourier transformed) Jacobian matrix is used. The algorithm is based on the approach of Bonani, Ghione, Pinto and Smith reported in [2] and extended to be used in the mixed-mode framework of Dessis$_{ISE}$ thereby taking all the different contact boundary conditions into account.
We describe the algorithm for the situation of one noisy physical device furnished with a spatial discretization grid of N internal vertices. The discretized (Fourier transformed) equation (3) has then the form

$$\begin{pmatrix} A & U \\ L & S \end{pmatrix} \begin{pmatrix} \delta x \\ \delta y \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \end{pmatrix} \tag{22}$$

where the complex matrix A is the Jacobian of the internal device equations wrt the internal variables, U the coupling of these equations to the circuit variables. S and L are the Jacobians of the circuit resp. boundary condition equations wrt the circuit/boundary resp. the internal device variables. The rhs $p := \left(p_x p_y \right)^T$ represents the discretized δ-function at one vertex of the grid, and $\delta u := \left(\delta x \delta y \right)^T$ is the perturbated solution splitted into the internal part δx and the boundary and circuit part δy.
To compute the potential Green functions wrt perturbations in one continuity equation for one observation node we have to solve the given system for N different rhs corresponding to the discretized δ-functions in all grid vertices. This can be written in the matrix form

$$\begin{pmatrix} A & U \\ L & S \end{pmatrix} \begin{pmatrix} \delta X \\ \delta Y \end{pmatrix} = \begin{pmatrix} P_x \\ P_y \end{pmatrix} \tag{23}$$

where $P := \begin{pmatrix} P_x \\ P_y \end{pmatrix}$ has N columns, each representing a discretized δ-function at one grid point. Using a blocked decomposition method the numerical burden can be drastically reduced. Instead of solving the complete linear system one reduces the system due to the fact that one is only interested in certain values of δy. The Schur decomposition of the matrix results in the equation

$$\begin{pmatrix} 1 & A^{-1}U \\ 0 & S - LA^{-1}U \end{pmatrix} \begin{pmatrix} \delta X \\ \delta Y \end{pmatrix} = \begin{pmatrix} A^{-1}P_x \\ P_y - LA^{-1}P_x \end{pmatrix} \tag{24}$$

or looking only onto the second component

$$\left(S - LA^{-1}U\right)\delta Y = P_y - LA^{-1}P_x \tag{25}$$

With the substitution $Y^T{:=}LA^{-1}$ we solve the equation

$$A^T Y = L^T \tag{26}$$

followed by the solution of the reduced equation (25). The Green function for a circuit node i is then given by

$$\widehat{G^i}(x_j) = \delta Y_{ij} \qquad . \tag{27}$$

Suitable choices of the perturbation matrix P give the necessary potential Green function$\widehat{G_n}$ and $\widehat{G_p}$ wrt both continuity equations for each observation node.
The same procedure is performed in the case of mixed-mode simulations with several physical noisy devices. In this situation the matrix A is block diagonal, i.e.

$$A = \begin{pmatrix} A_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & A_n \end{pmatrix} \tag{28}$$

and the perturbation matrix P is extended by discretizations of δ-functions for all grid points of all noisy devices.
The whole algorithm is interfaced to both the direct linear solvers Pardiso$_{ISE}$ and Super$_{ISE}$ and the iterative solver Slip$_{ISE}$ of Dessis$_{ISE}$. Besides the real extended formulation of the linear equations also the complex formulation can be assembled. This allows to put the complex mode of Pardiso$_{ISE}$ into action, which is most efficiently used in the case of up to several thousand vertices.

6 Noise Simulation with Dessis$_{ISE}$

6.1 General Remarks

The noise analysis capabilities in Dessis$_{ISE}$ can be used by selecting observation nodes (via the keyword *ObservationNode* within an *ACCoupled* solve-statement) and several noise source models in the physics-sections of the physical devices (via the keyword *Noise*).

6.2 Noisy ACCoupled

The observation nodes are selected in an *ACCoupled* solve statement, where also the noise extraction file and the fileprefix for noise plots are specified. A typical noisy *ACCoupled* looks like the following example:

```
ACCoupled (
  StartFrequency = 1.e8 EndFrequency 1.e11
  NumberOfPoints = 7 Decade
  Node ( n_drain n_gate )
  Exclude ( v_drain v_gate )
  ObservationNode ( n_drain n_gate )
  ACExtraction = "mos"
  NoiseExtraction = "mos"
  NoisePlot = "mos"
  ) {
    poisson electron hole contact circuit
}
```

The keyword *ObservationNode* enables the noise analysis. Please observe that in the current implementation the observation nodes have to be a subset of the nodes specified in *Node (...)*.
The noise auto- and cross-correlation spectral densities for all observation nodes are plotted into the file

<noise-extraction>_noise_des.plt .

If not specified the default "noiseextraction_noise_des.plt" is used.
The <noise-plot> string serves as a prefix for device specific plots and refers to the *NoisePlot* section for each device (see 6.5). For the auto-correlation noise data the filenames

<noise-plot>_<device-name>_<ob-node>_<number>_acgf_des.dat

are created while for the cross-correlation noise data the filenames

<noise-plot>_<device-name>_<ob-node-1>_<ob-node-2>_<number>_acgf_des.dat

are used, where <ob-node> is the observation node name, and the <number> is increased for each computed frequency. The default is to write these files compressed (extension *.Z*). This can explicitly be specified or switched off with the keyword *[-] Compressed*.
The selection of the frequencies for which the noise analysis is performed is like in the ac analysis. With the *Exclude* statement the type of boundary conditions for non observation nodes can be modified. For the observation nodes the strong couplings to voltage sources is "excluded" because otherwise the observed noise voltage is zero.

6.3 Green Functions

The most cpu consuming part within the noise analysis is the computation of the complex (Fourier transformed) Green functions. For the currently implemented noise source models only the Poisson Green function wrt perturbations in both continuity equations at the specified observation nodes are of interest, i.e. $\widehat{G_n^{\psi}}$ and $\widehat{G_p^{\psi}}$.
The real and imaginary (Re/Im) part of the complex potential (Po) Green functions wrt perturbations in the electron continuity (EC) equation $\widehat{G_n^{\psi}}$ can be plotted into the auto-correlation noise plot file as

PoECReACGreenFunction and *PoECImACGreenFunction*

for all devices and given observation nodes. The terms

GradPoECReACGreenFunction and *GradPoECImACGreenFunction*

refer to the corresponding parts of the vector Green function $\underline{\widehat{G_n^{\psi}}}$, while

Grad2PoECACGreenFunction

refers to the absolute square of $\underline{\widehat{G_n^{\psi}}}$. Corresponding expressions are valid for perturbations in the hole continuity (HC) equation.

6.4 Noise Sources

The noise sources of section 3 can be enabled for all physical devices in the physics section of the Dessis$_{-ISE}$ -command file

```
Physics {
    ...
    Noise ( DiffusionNoise MonopolarGRNoise FlickerGRNoise )
}
```

and specified like the other physical parameters for regions resp. for materials. The standard inheritance of physics parameters applies.

6.4.1 Diffusion Noise

The implemented diffusion noise source *DiffusionNoise* uses the Einstein relation $D = U_T \mu$ in equation (15) and results for electrons in the expression

$$K^{diff} = 4qnk_B T \mu_n \tag{29}$$

for the term on the diagonal, where n is the electron density, μ_n the electron mobility, and T the lattice or electron temperature. The corresponding expression is used for holes. With the specification

```
    DiffusionNoise ( <temp> )
```

where $<$*temp*$>$ is one of *LatticeTemperature*, *eTemperature*, *hTemperature*, or $e\_h\_$*Temperature*, the used temperature in the expression can be chosen. Default is *LatticeTemperature*. For example *eTemperature* uses the electron temperature for the the electron noise source while the lattice temperature is used for the hole noise source; the specification $e\_h\_$*Temperature* uses for the carrier noise source the corresponding carrier temperature.

6.4.2 Generation-Recombination (GR) Noise

Equivalent Monopolar GR Noise Source

The equivalent monopolar GR noise source model of equation (16) is called *MonopolarGRNoise* and the diagonal entries have the form

$$K^{GR}(x,f) = \frac{J_n J_n}{n} \frac{4\alpha\tau_{eq}}{1+\omega^2{\tau_{eq}}^2} \tag{30}$$

where J_n is the electron current density, n the electron density, α a fitting parameter, and τ_{eq} an equivalent GR lifetime. The parameters τ_{eq} and α can be modified for both carriers in the parameter file of Dessis$_{-ISE}$.

Flicker GR-Noise

The flicker GR noise model *FlickerGRNoise* for electrons (similar for holes) has the diagonal entries

$$K^{fGR}(x,f) = \frac{J_n J_n}{n} \frac{2\alpha_H}{\pi f \ln(\tau_1/\tau_0)} \left(arctan(\omega\tau_1) - arctan(\omega\tau_0)\right) \tag{31}$$

where J_n is the electron current density, n the electron density, α_H is a parameter, $\omega = 2\pi f$, and the time constants fulfill $\tau_0 < \tau_1$. The parameters α_H, τ_0, and τ_1 for electrons and holes are accessible via the parameter file of Dessis$_{-ISE}$.

6.5 Device Noise Data

Several variables can be plotted during the noise analysis. For each device one can specify a *NoisePlot* section similar to the *Plot* section, where the plotted data are listed. Besides the standard data you can specify additional noise specific data or groups of data listed in table 1 for the device auto-correlation data and in table 2 for the device cross-correlation data. We use the abbreviations LNS for local noise source and LNVSD for local noise voltage spectral density.
The currently implemented models result in the following expression for the the noise voltage spectral density

$$S_{\psi,\psi}(x,x';\omega) \tag{32}$$

$$= \int_\Omega \underline{\widehat{G_n^\psi}}(x,x_1;\omega)\underline{\underline{K}}_{n,n}^{Diff}(x_1;\omega)\underline{\widehat{G_n^\psi}}^*(x',x_1;\omega)dx_1 \tag{33}$$

$$+ \int_\Omega \underline{\widehat{G_p^\psi}}(x,x_1;\omega)\underline{\underline{K}}_{p,p}^{Diff}(x_1;\omega)\underline{\widehat{G_p^\psi}}^*(x',x_1;\omega)dx_1 \tag{34}$$

$$+ \int_\Omega \underline{\widehat{G_n^\psi}}(x,x_1;\omega)\underline{\underline{K}}_{n,n}^{GR}(x_1;\omega)\underline{\widehat{G_n^\psi}}^*(x',x_1;\omega)dx_1 \tag{35}$$

$$+ \int_\Omega \underline{\widehat{G_p^\psi}}(x,x_1;\omega)\underline{\underline{K}}_{p,p}^{GR}(x_1;\omega)\underline{\widehat{G_p^\psi}}^*(x',x_1;\omega)dx_1 \tag{36}$$

$$+ \int_\Omega \underline{\widehat{G_n^\psi}}(x,x_1;\omega)\underline{\underline{K}}_{n,n}^{fGR}(x_1;\omega)\underline{\widehat{G_n^\psi}}^*(x',x_1;\omega)dx_1 \tag{37}$$

$$+ \int_\Omega \underline{\widehat{G_p^\psi}}(x,x_1;\omega)\underline{\underline{K}}_{p,p}^{fGR}(x_1;\omega)\underline{\widehat{G_p^\psi}}^*(x',x_1;\omega)dx_1 \tag{38}$$

where all the noise sources $\underline{\underline{K}}$ are local current density noise sources. Each implemented noise source $\underline{\underline{K}}$ can be represented by one single real scalar quantity.

keyword	description	see equation
eeDiffusionLNS	electron/hole diffusion LNS	(33)
hhDiffusionLNS		(34)
eeMonopolarGRLNS	electron/hole monopolar GR LNS	(35)
hhMonopolarGRLNS		(36)
eeFlickerGRLNS	electron/hole flicker GR LNS	(37)
hhFlickerGRLNS		(38)
LNVSD	total LNVSD	(33) - (38)
eeLNVSD	total electron/hole LNVSD	(33), (35), (37)
hhLNVSD		(34), (36), (38)
eeDiffusionLNVSD	electron/hole diffusion LNVSD	(33)
hhDiffusionLNVSD		(34)
eeMonopolarGRLNVSD	electron/hole monopolar GR LNVSD	(35)
hhMonopolarGRLNVSD		(36)
eeFlickerGRLNVSD	electron/hole flicker GR LNVSD	(37)
hhFlickerGRLNVSD		(38)
PoECReACGreenFunction	real/imaginary (Re/Im) part of the	
PoECImACGreenFunction	potential (Po) Green function caused	
PoHCReACGreenFunction	by perturbations of the electron/hole	
PoHCImACGreenFunction	continuity (EC/HC) equation	
GradPoECReACGreenFunction	gradient of indicated Green function	
GradPoECImACGreenFunction		
GradPoHCReACGreenFunction		
GradPoHCImACGreenFunction		
Grad2PoECACGreenFunction	square of absolute value of	
Grad2PoHCACGreenFunction	indicated Green function	
AllLNS	all used LNS	
AllLNVSD	all used LNVSD	
GreenFunctions	Green functions and their gradients	

Table 1: Device Noise Data for Node Auto-Correlation

Auto-Correlation Data

The auto-correlation data refer to equation (32) where x and x' are identical. In this case the expression *eeDiffusionLNVSD* means the integrand of equation (33) where *ee* refers to the lower indices of the Green functions and *Diffusion* to the noise source. Similar the other integrands are named. The auto-correlation LNVSD, the LNS, and the Green function datasets are plotted for each device and observation node at given frequency into one file (see section 6.2) if they are selected in the *NoisePlot* section of the device; they are listed in table 1.

keyword	description
ReLNVXVSD *ImLNVXVSD*	re/im part of total cross LNVSD
ReeeLNVXVSD *ImeeLNVXVSD* *RehhLNVXVSD* *ImhhLNVSD*	re/im part of e/h cross LNVSD
ReeeDiffusionLNVXVSD *ImeeDiffusionLNVXVSD* *RehhDiffusionLNVXVSD* *ImhhDiffusionLNVSD*	re/im part of e/h diffusion cross LNVSD
ReeeMonopolarGRLNVXVSD *ImeeMonopolarGRLNVXVSD* *RehhMonopolarGRLNVXVSD* *ImhhMonopolarGRLNVSD*	re/im part of e/h monopolar GR cross LNVSD
ReeeFlickerGRLNVXVSD *ImeeFlickerGRLNVXVSD* *RehhFlickerGRLNVXVSD* *ImhhFlickerGRLNVSD*	re/im part of e/h flicker GR cross LNVSD
AllLNVXVSD	all used LNVXVSD

Table 2: Device Noise Data for Node Cross-Correlation

Cross-Correlation Data

In the case of $x \neq x'$ the node cross-correlation spectra are computed and the integrands become complex. *ReeeDiffusionLNVXVSD* and *ImeeDiffusionLNVXVSD* refer to the real resp. imaginary part of the integrand of equation (33). A list of the node cross-correlation device data is given in table 2. The cross-correlation LNVSD are plotted for each device and pair of observation nodes at given frequency into one file (see section 6.2) if they are selected in the *NoisePlot* section of the device; they are listed in table 2.

6.6 Node Noise Data

The noise analysis extracts for all given observation nodes the basic noise data and for all pairs of observation nodes the basic cross noise data. For each *ACCoupled* one file is generated according the specification <noise-extraction> (see section 6.2). Table 3 lists all the data which are plotted for each observation node, while table 4 lists all the possible cross noise data. The noise plot file can be postprocessed to derive noise figures NF and NFmin (and additional features).

keyword	description
S_V	noise voltage spectral density (NVSD)
S_V_ee	electron/hole NVSD
S_V_hh	
S_V_eeDiff	electron/hole NVSD
S_V_hhDiff	due to diffusion LNS
S_V_eeMonoGR	electron/hole NVSD
S_V_hhMonoGR	due to monopolar GR LNS
S_V_eeFlickerGR	electron/hole NVSD
S_V_hhFlickerGR	due to flicker GR LNS

Table 3: Node Auto-Correlation Data

keyword	description
ReS_VXV	re/im part of the cross noise voltage
ImS_VXV	spectral density (NVXVSD)
ReS_VXV_ee	re/im part of the
ImS_VXV_ee	electron/hole NVXVSD
ReS_VXV_hh	
ImS_VXV_hh	
ReS_VXV_eeDiff	re/im part of the
ImS_VXV_eeDiff	electron/hole NVXVSD
ReS_VXV_hhDiff	due to diffusion LNS
ImS_VXV_hhDiff	
ReS_VXV_eeMonoGR	re/im part of the
ImS_VXV_eeMonoGR	electron/hole NVXVSD
ReS_VXV_hhMonoGR	due to monopolar GR LNS
ImS_VXV_hhMonoGR	
ReS_VXV_eeFlickerGR	re/im part of the
ImS_VXV_eeFlickerGR	electron/hole NVXVSD
ReS_VXV_hhFlickerGR	due to flicker GR LNS
ImS_VXV_hhFlickerGR	

Table 4: Node Cross-Correlation Data

6.7 Noise Figure

For two-port devices the noise figure NF can be extracted as well as minimized wrt to the input admittance, resulting in the mimimum noise figure $NFmin$ and the optimized value Y_{min}.
The extraction is based on a single device simulation to extract both the Y-parameters of the two-port and the open-circuit equivalent noise voltage auto- and cross-correlation spectra for the input and output nodes. An example simulation for a MOSFET is given in Appendix C. With the specified node list the Y-parameters are extracted under the condition of grounded source and substrate nodes, while for the input (gate) and output (drain) the appropriate boundary conditions are imposed. The specification for the observation nodes allows the computation of the auto- and cross-correlation spectra of the equivalent open-circuit noise voltages at input and output node.
The noise figure extraction script (see Appendix D) performs the computation of the auto- and cross-correlation noise current spectra S_I^1, S_I^2, and S_I^{21}, the noise figure NF, the optimized input admittance Y_{min}, and the minimum noise figure $NFmin$. The user has to adjust the extraction script to the actual simulation:

names of the actual ac and noise simulation files

names of the input and output nodes

selection of the displayed curves.

With the command

```
inspect -f nf_ins.cmd
```

Inspect$_{ISE}$ is invoked and displays the selected curves. Observe that the sequence of the creation of curves should not be changed because the computation of the curves (partially) depends on the existence of preceding ones.

A Uniformly Doped Resistor

Here we give a command file example of a 1d simulation of a uniformly doped resistor:

```
Device "res100" {
  Electrode {
    { Name = "left"  Voltage = 0 resistor = 1. areafactor = 1.e8 }
    { Name = "right" Voltage = 0 resistor = 1. areafactor = 1.e8 }
  }
  File {
    Grid   = "1d_nres_100u_msh"
    Doping = "1d_nres_100u_msh"
  }
  Physics {
    Mobility ( DopingDep )
    Recombination ( SRH(DopingDep) Auger )
    Noise ( DiffusionNoise MonopolarGRNoise FlickerGRNoise )
  }
  NoisePlot {
    AllLNS AllLNVSD GreenFunctions
  }
}
System {
  res100 "res" ("left" = n_left "right" = n_right)
  v v_left  (n_left  0) { type="dc" dc=1.e-3 }
  v v_right (n_right 0) { type="dc" dc=0. }
}
Solve {
  Poisson
  Coupled { Poisson Electron Hole }
  ACCoupled (
    StartFrequency=1e-10 EndFrequency=1e20
    NumberOfPoints=31 Decade
    Node ( n_left n_right )
    Exclude ( v_left )
    ObservationNode ( n_left )
    acextraction="1d_difm"
    noiseextract="1d_difm"
    noiseplot="1d_difm"
    ) {
      Poisson Electron Hole
  }
}
```

B Parameter File

Here we give the noise section of the parameter file of Dessis$_{ISE}$:

```
MonopolarGRNoise
{
 *-----------------------------------------------------------------*
 *      K = |J_n|^2/n * (4 e_alpha e_tau)/(1 + omega^2 e_tau^2)    *
 *-----------------------------------------------------------------*
 * with J_n electron current density, n electron density.          *
 * Corresponding expression for holes                              *
 *-----------------------------------------------------------------*
        e_alpha = 1 # [1]
        h_alpha = 1 # [1]
        e_tau = 1.0000e-07 # [s]
        h_tau = 1.0000e-07 # [s]
}

FlickerGRNoise
{
 *-----------------------------------------------------------------*
 *      K = |J_n|^2/n * (2 e_alpha_H)/(pi f ln(e_tau1/e_tau0))     *
 *            * ( arctan(omega e_tau1) - arctan (omega e_tau0) )   *
 *-----------------------------------------------------------------*
 * with J_n electron current density, n electron density,          *
 *   f frequency, omega = 2 pi f .                                 *
 * Corresponding expression for holes                              *
 *-----------------------------------------------------------------*
        e_alpha_H = 2.0000e-03 # [1]
        h_alpha_H = 2.0000e-03 # [1]
        e_tau0 = 1.0000e-06 # [s]
        h_tau0 = 1.0000e-06 # [s]
        e_tau1 = 3.0000e-04 # [s]
        h_tau1 = 3.0000e-04 # [s]
}
```

C NF Simulation of a MOSFET

Here we give an example for a HF simulation of a MOSFET suitable for a noise figure extraction:

```
Device "nmos" {
  Electrode {
   { name=source voltage=0. resistance=1. AreaFactor=200 }
   { name=gate   voltage=0. resistance=1. AreaFactor=200
                 barrier=-0.45}
   { name=drain  voltage=0. resistance=1. AreaFactor=200 }
   { name=bulk   voltage=0. resistance=1. AreaFactor=200 }
  }
  Physics {
    Recombination ( SRH(DopingDep) Auger Avalanche(Lackner) )
    Mobility ( DopingDep Enormal HighFieldSaturation )
    Noise ( DiffusionNoise )
  }
  InterfaceConditions {
    { region = (0,1) RecombVelocity = 500 Charge = 4e10 }
  }
  File {
    Grid    = "n21_mdr.grd"
    Doping  = "n21_mdr.dat"
    param   = "nmos.par"
  }
}

Math {
  Method = Blocked
  Submethod = Pardiso
  Derivatives
  AvalDerivatives
  NewDiscretization
}

System {
  nmos "NMOS" ( "source" = nsource "gate" = ngate
                "drain"  = ndrain  "bulk" = nbulk )

  v vgate   ( ngate   0 ) { type="dc" dc=0. }
  v vsource ( nsource 0 ) { type="dc" dc=0. }
  v vbulk   ( nbulk   0 ) { type="dc" dc=0. }
  v vdrain  ( ndrain  0 ) { type="dc" dc=0. }
}
```

```
Solve {
  load ( fileprefix = "save/init_dd_080Vg_250Vd" )

  ACCoupled (
    StartFrequency=1e8 EndFrequency=1e11
    NumberOfPoints=16 Decade
    Node ( ndrain ngate )
    Exclude ( vdrain vgate )
    ObservationNode ( ndrain ngate )
    ACExtraction = "dd_080Vg_250Vd"
    Noiseextract = "dd_080Vg_250Vd"
    NoisePlot    = "noiseplot/dd_080Vg_250Vd"
    ) {
      poisson electron hole circuit contact
  }
}

NoisePlot {
  AllLNS AllLNVSD AllLNVXVSD
}
```

D Noise Figure Extraction Script

Here we give the noise figure extraction script ‘nf_ins.cmd‘:

```
#-------------------------------------------------------------------
# INITIALIZATION
#-------------------------------------------------------------------

load_library sms

# global variables
global SMS_c_project_ac

# local variable
set cv_list [list]

#-------------------------------------------------------------------
# USER INPUT
#-------------------------------------------------------------------

# --- USER FILES
SMS_file_ac dd_080Vg_250Vd_ac_des.plt
SMS_file_noise dd_080Vg_250Vd_noise_des.plt

# --- USER NODES
set ninput "ngate"
set noutput "ndrain"

SMS_dut_input $ninput
SMS_dut_output $noutput

# --- NOISE SOURCE
# impedance Zs = (ReZs , ImZs) in Ohm
SMS_Zs 50. 0.

# --- OTHER VARIABLES
SMS_logfile nf_ins.log

#-------------------------------------------------------------------
# STARTING SMS MODUL
#-------------------------------------------------------------------

SMS_start
```

```
#--------------------------------------------------------------------
# COMPUTING HELP CURVES
#--------------------------------------------------------------------

# --- frequency -
set cv_list [SMS_create_frequency $SMS_c_project_ac "frequency"]
#SMS_display $cv_list

# --- conductances a and capacitances c
set cv_list [SMS_create2 $SMS_c_project_ac frequency \
     $ninput $ninput {a c}]
#SMS_display $cv_list
set cv_list [SMS_create2 $SMS_c_project_ac frequency \
     $noutput $noutput {a c}]
#SMS_display $cv_list
set cv_list [SMS_create2 $SMS_c_project_ac frequency \
     $ninput $noutput {a c}]
#SMS_display $cv_list
set cv_list [SMS_create2 $SMS_c_project_ac frequency \
     $noutput $ninput {a c}]
#SMS_display $cv_list

# --- Y-parameters
set cv_list [SMS_create_Y_from_DESSIS $ninput $ninput]
#SMS_display $cv_list
set cv_list [SMS_create_Y_from_DESSIS $noutput $noutput]
#SMS_display $cv_list
set cv_list [SMS_create_Y_from_DESSIS $ninput $noutput]
#SMS_display $cv_list
set cv_list [SMS_create_Y_from_DESSIS $noutput $ninput]
#SMS_display $cv_list

# --- Z-parameters (not necessary for following computations)
set cv_list [SMS_create_Z_from_Y $ninput $ninput]
#SMS_display $cv_list
set cv_list [SMS_create_Z_from_Y $noutput $noutput]
#SMS_display $cv_list
set cv_list [SMS_create_Z_from_Y $ninput $noutput]
#SMS_display $cv_list
set cv_list [SMS_create_Z_from_Y $noutput $ninput]
#SMS_display $cv_list

# --- noise voltage spectra S_V
set cv_list [SMS_create_S_V $ninput $noutput]
SMS_display $cv_list
```

```
# --- noise current spectra S_I
set cv_list [SMS_create_S_I $ninput $noutput]
SMS_display $cv_list

#-------------------------------------------------------------------
# NOISE FIGURES NF and NFmin and OPTIMIZED INPUT ADMITTANCE Ymin
#-------------------------------------------------------------------
# you can choose one (or more) of following branches
#   (A) NF computation
#   (B) NFmin computation
#-------------------------------------------------------------------

# --- (A) NF computation -
set cv_list [SMS_create_NF $ninput $noutput]
SMS_display $cv_list

set cv_list [SMS_create_NF_db20 $ninput $noutput]
#SMS_display $cv_list

#-------------------------------------------------------------------

# --- (B) NFmin computation -
set cv_list [SMS_create_Ymin $ninput $noutput]
#SMS_display $cv_list

set cv_list [SMS_create_NFmin $ninput $noutput]
SMS_display $cv_list

set cv_list [SMS_create_NFmin_db20 $ninput $noutput]
#SMS_display $cv_list

#-------------------------------------------------------------------
# END OF SMS MODUL
#-------------------------------------------------------------------
```

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