An improved approach to the Shockley–Read–Hall recombination in inhomogeneous fields of space-charge regions

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The rate formula of Shockley–Read–Hall (SRH) recombination is generalized for multiphonon transitions in an inhomogeneous electric field. A three-band model and Fermi statistics are used. The thermal generation and recombination at deep centers is tunnel assisted, if it occurs in the electric field of space-charge layers. Trap tunneling without phonon assistance turns out to be an exceptional case as well as the exclusive thermal multiphonon transition at zero field strength. Generally, every single recombination act is a combination of thermal capture and tunneling into the trap. A background of the order of an interband tunnel length contributes to the local rate. In the Boltzmann case the well known form of the SRH rate is derived, but with spatially variable lifetimes. Their position dependence directly reflects the shape of the electric field strength, if the traps are uniformly distributed. Depending on the external parameters, the calculation of the SRH current density leads to *I-U* characteristics, which exhibit interesting substructures in the case of certain discrete trap configurations. The theory is applied to a $Hg_{0.8}Cd_{0.2}Te$ n^+p junction and a GaAs surface potential.

I. INTRODUCTION

Since the pioneering work of Shockley, Read, and Hall[1] on the statistics of the recombination of electrons and holes many papers have been published aiming at the improvement of theoretical understanding of these processes, which play an outstanding role in semiconductor devices.

Sah *et al.*[2] calculated the recombination current by analytical integration of the recombination rate over a depleted space-charge region assuming a linear potential there. In 1958 deep level-to-band tunneling transitions were found to be a source of excess current in tunnel diodes.[3] Price[4] and Sah[5] gave expressions for the Wentzel–Kramers–Brillouin (WKB) probability of these processes and an estimate of the transition matrix element. In a series of papers Sah[6] (and references therein) developed equivalent circuit models for thermal, optical, Auger-impact, and tunneling recombination-generation-trapping processes. In Ref. 6 he considered traps being located in the wide gap material of a heterojunction, making possible elastic and inelastic tunneling transitions into the small gap material.

After the interest in tunnel diodes had disappeared, a revival of impurity-to-band tunneling calculations began in 1980 in connection with junction currents in narrow gap semiconductors and efficiency limiting processes in solar cells. Wong[7] considered the traps being located near the space-charge layer but not within it, which resulted in a model with decoupling of tunneling and thermal recombination. Anderson and Hoffman[8] calculated the current of a $Hg_{1-x}Cd_xTe$ *pn* junction using the Kane $E(k)$ dispersion relation combined with a parabolic barrier field. They determined the transition matrix element using a Coulombic ground state for the deep level wave function, and they obtained the total current by simply adding the thermal and the tunnel part neglecting the interdependence of both via the occupation of the deep center.

In Ref. 8 the result for the tunneling probability in such systems was slightly improved taking the exact band wave functions of a four-band model. Plazcek-Popko and Pawlikowski[10] reported a forward characteristic of a HgCdTe diode at $T < 20$ K with three minima in its reciprocal differentiated form ($r_d - U$ characteristic), which they interpreted as phonon-assisted tunneling. A similar $r_d - U$ curve (with two minima) was obtained recently by Heukenkamp[11] for a n^+p diode of $Hg_{0.8}Cd_{0.2}Te$ at 11 K. He attributed the minima to two different trap levels at $E_g/4$ and $E_g/2$ and showed that the minima are quenched by a magnetic field of 4.4 T. For the fit he used an improved version of the model of Anderson and Hoffman[8] including the combination of thermal and tunnel traffic via the trap occupancy.

In 1989 Waterman *et al.*[12] observed trap-assisted tunneling to quantized electric subbands in $Hg_{0.78}Cd_{0.22}Te$ metal-insulator-semiconductor (MIS) capacitors, which they described as a two-step process—thermal capture of a valence electron and following tunneling into the subband. It appears to be a drawback of the current calculations in Refs. 8 and 11 that only two recombination paths were taken into account: a thermal (vertical) and a tunnel (horizontal) transition. Both paths are actually very improbable: the first dominates for zero electric field, the latter for zero temperature. On the other hand from a theoretical

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point of view, the separation of the two channels is artificial, since the trap tunneling transition is only a special Shockley–Read–Hall (SRH) recombination path with no phonon assistance at all.

In this paper the SRH recombination is generalized to the inhomogeneous electric fields of space-charge layers by equating the vertical transitions with nonradiative multiphonon capture and emission processes of electrons and holes. Each multiphonon step can be tunnel assisted, if there are field induced states near the band edges at the trap position or in the picture of tilted bands, if there are band-edge states at the energy of the multiphonon step in a distance of the tunneling length.

The recombination statistics will be briefly reviewed in Sec. II, the problems of capture rates, density of states in inhomogeneous fields, and the resulting total SRH rate will be taken up in the following three sections. In Sec. VI some applications in the case of Boltzmann statistics will be discussed: analytical expressions for spatially variable SRH lifetimes and the trap tunneling current. The general rate formula will be studied in Sec. VII for uniform and discrete trap configurations. Some calculated device characteristics of a $Hg_{0.8}Cd_{0.2}Te$ n^+p junction and a GaAs surface potential will be presented. The final section will give a brief discussion of the results and a summary.

II. ENERGETIC SRH RATES

The density of energy states in a semiconductor is changed by the presence of an electric field. New states, the so called Franz–Keldysh tails,[13] appear in the gap and give rise to tunneling transitions. The continuous nature of the energy spectrum requires that the description of the generation-recombination kinetics includes such new states. In doing so we limit ourselves to the conventional picture of the nonequilibrium situation, which is characterized by two separate Fermi levels—for the conduction electrons and for the holes. Thus, we assume that the recombination time greatly exceeds the mean free path time, although low temperatures and the electric field of the space-charge region restrict this conception.[14] Furthermore, we suppose one-level recombination centers of the same kind having a thermal binding energy E_t measured from the edge of the conduction band $E_c(x) = E_c^0 - e\varphi(x)$ [x: field direction, $\varphi(x)$: electrostatic potential]. For clearness we denote energies above the trap level by E' and below by E.

The differential net recombination rate of electrons and holes

$$dR_{n,p} = dr_{n,p} - dg_{n,p} \tag{1}$$

follows from the differential recombination and generation rates:

$$\begin{aligned} dr_n &= N_t(1-f_t)c_n(E')N_c(E')f_c(E')dE', \\ dg_n &= N_t f_t e_n(E')N_c(E')[1-f_c(E')]dE', \end{aligned} \tag{2}$$

and

$$\begin{aligned} dr_p &= N_t f_t [c_{\rm hh}(E)N_{\rm hh}(E) + c_{\rm lh}(E)N_{\rm lh}(E)] \\ &\quad \times [1-f_v(E)]dE, \\ dg_p &= N_t(1-f_t)[e_{\rm hh}(E)N_{\rm hh}(E) \\ &\quad + e_{\rm lh}(E)N_{\rm lh}(E)]f_v(E)dE. \end{aligned} \tag{3}$$

In (2) and (3) N_t is the trap density, f_t the trap occupation probability, and $c_i(E)$ and $e_i(E)$ are the spectral capture and emission rates. $N_i(E)$ denotes the energetic densities of states, and $f_{e,v}(E)$ are the Fermi functions of conduction and valence band. Heavy and light holes are described by a joint Fermi level F_p, but by different capture and emission rates. In (3) the terms in braces will not be cut in the following calculation. Thus, we use $c_p(E)N_v(E)$ and $e_p(E)N_v(E)$ as abbreviations, respectively.

In thermodynamic equilibrium two relations of detailed balance follow from (2) and (3):

$$\begin{aligned} e_n(E') &= c_n(E')\frac{(1-f_t^0)f_0(E')}{f_t^0[1-f_0(E')]}, \\ e_p(E) &= c_p(E)\frac{f_t^0[1-f_0(E)]}{f_0(E)(1-f_t^0)}. \end{aligned} \tag{4}$$

In the stationary case $R_n = R_p$ holds. This gives an equation for the trap occupation probability f_t

$$(1-f_t)\hat{c}_n - f_t\hat{e}_n = f_t\hat{c}_p - (1-f_t)\hat{e}_p \tag{5}$$

with

$$\begin{aligned} \left.\begin{matrix}\hat{c}_n\\ \hat{e}_n\end{matrix}\right\} &= \int_{E_c-E_t}^{\infty} dE' N_c(E') \times \begin{cases} c_n(E')f_c(E') \\ e_n(E')[1-f_c(E')] \end{cases} \\ \left.\begin{matrix}\hat{c}_p\\ \hat{e}_p\end{matrix}\right\} &= \int_{-\infty}^{E_c-E_t} dE N_v(E) \times \begin{cases} c_p(E)[1-f_v(E)] \\ e_p(E)f_v(E) \end{cases}. \end{aligned} \tag{6}$$

The integrations in (6) include all gap states above and below the trap level, respectively, since a carrier can be captured from or be emitted into such states, which originate from the presence of an electric field.

If f_t is determined from (5) and then again inserted into the left-hand side of (5), the total net rate in the stationary case comes out:

$$R = N_t \frac{\hat{c}_p\hat{c}_n - \hat{e}_p\hat{e}_n}{\hat{c}_n + \hat{e}_n + \hat{c}_p + \hat{e}_p}. \tag{7}$$

Now, using (6) and expressing the emission rates by the capture rates via (4), one obtains

$$\begin{aligned} R &= \left[1 - \exp\left(-\frac{F_n(x) - F_p(x)}{kT}\right)\right] \int_{-\infty}^{\infty} dE' \\ &\quad \times \int_{-\infty}^{\infty} dE R(E',E) f_c(E')[1-f_v(E)]. \end{aligned} \tag{8}$$

By $R(E',E)$ we have introduced an energetic net SRH rate

$$R(E',E) = N_t \frac{N_c(E')c_n(E')\Theta(E'-E_c+E_t)N_v(E)c_p(E)\Theta(E_c-E_t-E)}{\hat{c}_n/f_t^n + \hat{c}_p/(1-f_t^p)} \tag{9}$$

with $\hat{c}_n$ and $\hat{c}_p$ given in (6).

f_t^n and f_t^p are trap occupation probabilities defined with the corresponding quasi-Fermi levels of electrons and holes:

$$f_t^\nu = \left[1 + \frac{g_{0,\nu}}{g_{1,\nu}} \exp\left(\frac{E_c - E_t - F_\nu}{kT}\right)\right]^{\mp 1}$$

(upper sign for electrons, $g_{0,1}$—degeneracy factors of the empty and occupied trap level, respectively).

Since the recombination at deep centers goes through two steps—capture of a conduction electron and capture of a hole—the rate $R(E',E)$ depends on both variables E' and E separately. Figure 1 illustrates the recombination of an electron-hole pair in the electric field of a space-charge region. Only one transition path via tail states has been marked. Obviously, a background of the order of an interband tunneling length contributes to the local rate at point x. The total rate R depends on x via the trap concentration $N_t(x)$, the electrostatic potential $\varphi(x)$, and the quasi-Fermi levels $F_n(x)$, $F_p(x)$. In heterostructures graded gap effects would come into play additionally.

III. SPECTRAL CAPTURE RATES

We consider the SRH process to be a multiphonon transition, where the electronic energy is transformed into lattice vibrations. Due to the strong localization of the bound state and the corresponding delocalization of the wave function in k space, momentum conservation is not required.

The calculation of zero field capture and emission rates of nonradiative multiphonon-transitions has been performed explicitly in various papers.[15–21] First Makram-Ebeid and Lannoo 1982[22] came along with a theory of multiphonon transitions in a homogeneous electric field. There they extended the theory of tunneling between gap and band states to the case where multiphonon transitions can take place in addition, i.e., where tunneling processes become phonon-assisted. A slight improvement was achieved in Ref. 23 by treating the problem as a field-enhanced multiphonon process, which is both thermally and field induced. In this treatment the electric field enters into the emission rate via the combined density of states. Therefore, the various field effects, like Franz–Keldysh effect, Stark effect, Poole–Frenkel effect, or field induced line shape broadening of the deep level can be included. Furthermore, the WKB approximation can be avoided, which is necessary if the transitions predominantly occur near the band edges.

In order to restrict the number of new parameters in the description of SRH recombination to a minimum, the following limitations are used: The deep level wave function is determined by the envelope method with Lucovski's δ-function potential[24] yieldig simple Koster–Slater orbitals. Only one effective phonon mode with energy $\hbar\omega_0$ for both kinds of carriers is assumed (Einstein model), and the frequency ω_0 is thought not to change during the transition. Band states do not couple to this mode. Then one obtains for the spectral capture rate of electrons[23]

$$c_n(E') = c_n^0\left(r_{F,n}^2 + \frac{(E_c - E_t + S\hbar\omega_0 - E')^2}{S(\hbar\omega_0)^2} r_{\text{ph},n}^2\right) \times L(E_c - E_t - E') \tag{10}$$

with the lineshape function of multiphonon theory

$$L(\epsilon) = 2\pi\hbar e^{-S(2f_B+1)} \sum_{l=-\infty}^{\infty} \left(\frac{f_B+1}{f_B}\right)^{l/2} \times I_l[2S\sqrt{f_B(f_B+1)}]\delta(l\hbar\omega_0 + \epsilon). \tag{11}$$

In (10) and (11) S is the Huang–Rhys factor, which is a measure of the coupling strength of the diagonal electron-phonon interaction, $S\hbar\omega_0$ defines the lattice relaxation energy, $r_{F,n}$ and $r_{\text{ph},n}$ are matrix elements of the nondiagonal electron-field ($r_{F,n}$) and electron-phonon ($r_{\text{ph},n}$) coupling, respectively. The prefactor c_n^0 contains various unknown quantities, like the impurity potential strength, the symmetry of the wave function, the localization radius a.o., so it will enter into the constant part of the electron lifetime $\tau_{n,0}$ finally.

The lineshape function depends on the phonon occupation number $f_B = [\exp(\hbar\omega_0/kT) - 1]^{-1}$ and represents a series of δ peaks weighted by modified Bessel functions I_l. The envelope of the spectrum is a Gaussian for high temperatures.

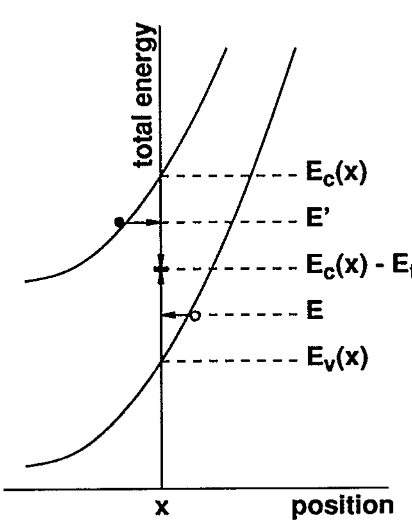

FIG. 1. One possible recombination path via a deep level at $E_c(x) - E_t$ in a space-charge region.

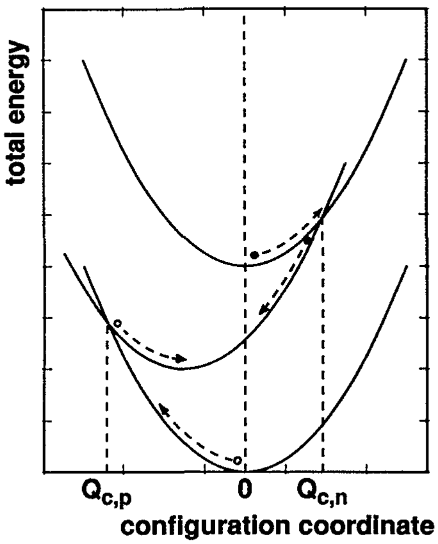

FIG. 2. Configuration-coordinate diagram with the crossing points of the harmonic potential parabolas at zero electric field. The two-band parabolas have to be replaced by a quasi-continuum of such parabolas, if the recombination takes place in an electric field. Then, both activation energies are lowered.

With (10) and (11) we find for the spectral capture rates of electrons and holes, respectively

$$c_n(E') = 2\pi\hbar c_n^0 \sum_{l>0} \left(r_{F,n}^2 + \frac{(l-S)^2}{S} r_{\mathrm{ph},n}^2 \right) \times L(l)\delta(l\hbar\omega_0 + E_c - E_t - E'),$$

$$c_p(E) = 2\pi\hbar c_p^0 \sum_{l>0} \left(r_{F,p}^2 + \frac{(l+S)^2}{S} r_{\mathrm{ph},p}^2 \right) \times L(l)\delta(l\hbar\omega_0 + E - E_c + E_t). \quad (12)$$

$L(l)$ involves all f_B-dependent factors of (11). Note, that c_p^0, $r_{\mathrm{ph},p}$ and $r_{F,p}$ are different for light and heavy holes. The physical meaning of the terms $(l \pm S)^2/S$ in (12) is easily understood. Let E_t be a midgap level and $l_0 = \mathrm{int}(E_t/\hbar\omega_0)$ the number of phonons emitted during the capture of an electron at zero electric field. Then, it is clear from a configuration-coordinate diagram (Fig. 2) that $(l_0 - S)/\sqrt{S}$ is just the crossing point $Q_{c,n}$ of the two adiabatic potential parabolas with the electronic energies E_c and $E_c - E_t$, respectively. In the same way, $Q_{c,p} = -(l_0 + S)/\sqrt{S}$ represents the crossing point of the two parabolas with electronic energies $E_c - E_t$ and E_v, respectively. The $Q_{c,i}$ appear in the capture rates as the squared "phononic part" of the electron-phonon interaction operator $QV(r)$.

For comparison with experiment it is reasonable to define only one constant lifetime $\tau_{i,0}$ for each kind of carriers. After (12) this can be done either by averaging the brackets ($l = l^*$) or by neglecting one of the transition matrix elements. An estimate of the ratio r_{ph}/r_F with usual models of the electron-phonon coupling favors r_{ph}, thus we neglect r_F in the further calculation.

IV. COMBINED DENSITY OF STATES IN INHOMOGENEOUS FIELDS

The electric field of a space-charge region is never constant. In band-to-band or band-to-trap tunneling calculations the assumption of a constant field is approximately fulfilled only in the case of strongly reversed biased junctions and ignoring the boundaries of the depletion layer. In order to find a better approximation for the local density of band states in inhomogeneous electric fields, we use a result for the envelope wave function in such fields, which was derived in Ref. 25. There it was shown that for medium varying potentials, i.e., for fields which actually vary over an interband tunneling length l_{int}, but which can still be linearized over a small portion Δl_{int} of this length

$$\Delta l_{\mathrm{int}} = 2\left(\frac{\hbar\Theta}{E_g}\right)^{3/4} l_{\mathrm{int}} \quad (13)$$

[$\Theta = (e^2F^2/2\mu\hbar)^{1/3}$, μ—reduced effective mass, F—field strength], the envelope of conduction-band states has the approximate form

$$f_{E',\mathbf{k}}^c(x) = \sqrt{\frac{2m_c}{\hbar^2}} \frac{\sqrt{\pi}}{\sqrt{|\kappa_{E',\mathbf{k}}^c(x)|}} \left|\tfrac{3}{2} S_{E',\mathbf{k}}^c(x)\right|^{1/6} \times \mathrm{Ai}[(\tfrac{3}{2} S_{E',\mathbf{k}}^c(x))^{2/3}]. \quad (14)$$

In (14) $S_{E',\mathbf{k}}^c(x)$ is the action integral

$$S_{E',\mathbf{k}}^c(x) = \int_{x_{E',\mathbf{k}}^c}^{x} dx' \kappa_{E',\mathbf{k}}^c(x') \quad (15)$$

with the wave number $\kappa_{E',\mathbf{k}}^c(x)$, which follows from

$$[\kappa_{E',\mathbf{k}}^c(x)]^2 = \begin{cases} k_\perp^2 + \dfrac{2m_c}{\hbar^2}[E_c(x) - E'] & \text{parabolic bands} \\ \dfrac{m_c}{\hbar^2}\left[\dfrac{2}{E_g}(E' - E_c(x) + E_g/2)^2 - E_g/2 - \dfrac{\hbar^2 k_\perp^2}{m_c}\right] & \text{Kane bands.} \end{cases} \quad (16)$$

The quasi-classical turning point $x_{E',\mathbf{k}}^c$ is solution of

$$\kappa_{E',\mathbf{k}}^c(x_{E',\mathbf{k}}^c) = 0. \quad (17)$$

Far from classical turning point the envelope (14) turns to the well known semiclassical limits, but in the vicinity of this point it can be developed up to first order, yielding the usual Airy solution.

Now, (14) can be used to find an approximate expression for the local density of states in inhomogeneous fields. Actually, the combined density of states turns out in form of a product of trap and band density of states, respectively, due to the strong localization of the impurity wave function. The trap density of states (here only one level) is a delta function, since the field induced line shape broadening was neglected. It does not appear explicitly, since the corresponding integration is already involved in the definition of the recombination rates in Eqs. (2) and (3). The remaining local density of conduction band states is given then by

$$N_c(E',x) = \frac{1}{4\pi^3} \int d^2\mathbf{k}_\perp |f^c_{E',\mathbf{k}}(x)|^2. \quad (18)$$

The integral is calculated neglecting the $\mathbf{k}_\perp$ dependence of the slowly varying factors of the envelope. If the argument of the Airy function is developed up to first order in $k_\perp^2$, one obtains

$$N_c(E',x) = \left(\frac{m_c}{\pi\hbar^2}\right) \frac{1}{\xi^c_{E'}(x)} [\tfrac{3}{2} S^c_{E',0}(x)]^{2/3} \mathscr{F}\{[\tfrac{3}{2} S^c_{E',0}(x)]^{2/3}\} \quad (19)$$

with

$$\xi^c_{E'}(x) = \frac{1}{2} \kappa^c_{E',0}(x) \int_{x^c_{E',0}}^{x} dx' [\kappa^c_{E',0}(x')]^{-1}. \quad (20)$$

In (19) $\mathscr{F}(y)$ is the electro-optical function $\mathscr{F}(y) = \mathrm{Ai}^2(y) - y\mathrm{Ai}'^2(y)$.[26] Equation (19) represents a generalization of the density of conduction-band states to an inhomogeneous field $F(x)$. The limit of a constant field is easily verified in the case of parabolic bands using

$$[\tfrac{3}{2} S^c_{E',0}(x)]^{2/3} \rightarrow \frac{E_c(x) - E'}{\hbar\Theta_c}, \quad \xi^c_{E'}(x) \rightarrow \frac{E_c(x) - E'}{eF(x)}. \quad (21)$$

Thus, the well known Franz–Keldysh result[13] follows:

$$N_c^{\mathrm{hom}}(E',x) = \frac{1}{2\pi} \left(\frac{2m_c}{\hbar^2}\right)^{3/2} \sqrt{\hbar\Theta_c} \mathscr{F}\left(\frac{E_c(x) - E'}{\hbar\Theta_c}\right). \quad (22)$$

For vanishing field strength (22) tends to

$$N_c^{F=0}(E',x) = \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2}\right)^{3/2} \sqrt{E' - E_c(x)} \times \Theta[E' - E_c(x)], \quad (23)$$

i.e., only states above the band edge contribute to the SRH rate in such regions.

Going through the same steps the corresponding expressions for light and heavy holes can be derived. In (14)–(23) the effective mass m_c has to be replaced by m_{hh} or m_{lh}, and $E_c(x) - E'$ by $E - E_v(x)$.

V. TOTAL SRH RATE

Up to here we have gathered all quantities, which are necessary to write down the total net SRH rate in the general case. Due to the δ-functions in the spectral capture rates all integrals reduce to sums over positive integers—the number of phonons created during the capture. Let $\rho_i(\epsilon)$ be dimensionless energetic densities of states

$$\rho_i(\epsilon) = \frac{E_t}{N_i^0} N_i(\epsilon,x). \quad (24)$$

N_i^0 denote the effective band densities of states ($i = c,\mathrm{lh},\mathrm{hh}$). Furthermore, instead of calculating the transition matrix elements explicitly, we introduce constants $\tau_{i,0}$ which do not depend on temperature or electric field strength and which serve as fit parameters. Then, the net SRH rate has the form

$$R(x) = N_t(x)\left[1 - \exp\left(-\frac{F_n(x) - F_p(x)}{kT}\right)\right] \times \frac{1}{\tau_p(x) + \tau_n(x)} \quad (25)$$

with

$$\tau_n^{-1}(x) = (1 - f_t^p) \times \sum_{l>0} \tau_{n0}^{-1} \rho_c(l) \frac{(l-S)^2}{S} L(l) f_c(l),$$

$$\tau_p^{-1}(x) = f_t^n \sum_{l>0} [\tau_{\mathrm{lh},0}^{-1} \rho_{\mathrm{lh}}(l) + \tau_{\mathrm{hh},0}^{-1} \rho_{\mathrm{hh}}(l)] \times \frac{(l+S)^2}{S} L(l) [1 - f_v(l)]. \quad (26)$$

The dimensionless densities of states $\rho_i(l)$ and the thermal distributions $L(l)$ are given by

$$\rho_i(l) = 2\sqrt{\pi} \frac{E_t}{kT} \frac{1}{k^c_{\mathrm{th}} \xi_c(l)} [\tfrac{3}{2} S^i_{E_l,0}(x)]^{2/3} \times \mathscr{F}\{[\tfrac{3}{2} S^i_{E_l,0}(x)]^{2/3}\}, \quad (27)$$

where

$$E_l = \begin{cases} l\hbar\omega_0 + E_c - E_t & \text{for electrons} \\ E_c - E_t - l\hbar\omega_0 & \text{for holes,} \end{cases}$$

and

$$L(l) = e^{-S(2f_B+1)} \left(\frac{f_B + 1}{f_B}\right)^{l/2} I_l[2S\sqrt{f_B(f_B+1)}]. \quad (28)$$

The quantity k^c_{th} denotes the thermal wave number $k^c_{\mathrm{th}} = (2m_c kT/\hbar^2)^{1/2}$. Equations (25)–(28) generalize the SRH rate to inhomogeneous fields, Fermi statistics, and three bands. Only two new parameters are needed—the Huang–Rhys factor S and the effective phonon energy $\hbar\omega_0$.

The transport of a carrier via a deep level proceeds on a combined way: tunneling to/from the trap position (horizontal part) and thermal capture/emission (vertical part). All ways on the "phonon ladder" E_l are possible, but there is of course a most probable way, i.e., a most probable number of phonons l^*, depending on temperature T, field strength F, thermal binding energy E_t, Huang–Rhys factor S, and phonon energy $\hbar\omega_0$. In general, l will be different for electrons and holes. An illustration is shown in Fig. 3.

The numerical expense in this theory reduces to the determination of the quasi-classical turning points, i.e., the crossing points of the phonon ladders with the band edges, and to the calculation of the tunneling probability (27) in regions with noticeable electric field. The thermal distribution $L(l)$ is the same in every point and has to be calculated only one time. The action integrals may be transformed into sums using the phonon ladder as natural grid, at least if $\hbar\omega_0 \ll E_g$ holds.

VI. APPLICATION TO BOLTZMANN STATISTICS

Now, we consider new definitions of averaged capture rates $\langle c_i \rangle$ with the dimension $cm^3\,s^{-1}$:

$$\tau_n^{-1}(x) = (1 - f_t^p) n \langle c_n \rangle$$
$$\tau_p^{-1}(x) = f_t^n \{ p_{lh} \langle c_{lh} \rangle + p_{hh} \langle c_{hh} \rangle \}. \tag{29}$$

The structure of the $\langle c_i \rangle$ is obvious from a comparison with (26).

In the case of Boltzmann statistics the rate formula (25) then can be written in the form

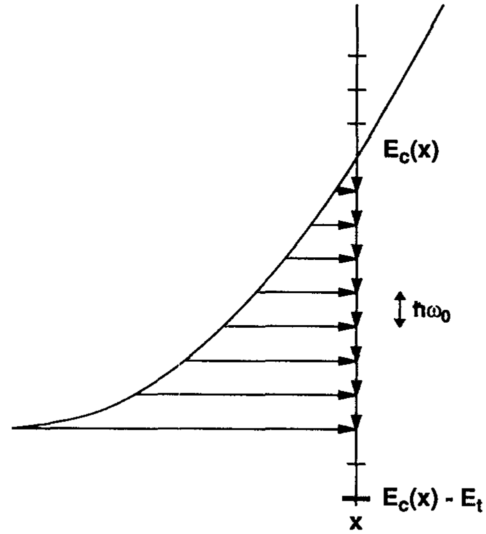

FIG. 3. Electron recombination paths: tunneling to the trap and subsequent thermal capture.

$$R(x) = N_t(x) \frac{\langle c_n \rangle \langle c_{lh} \rangle (np_{lh} - n_0 p_{lh,0}) + \langle c_n \rangle \langle c_{hh} \rangle (np_{hh} - n_0 p_{hh,0})}{\langle c_n \rangle (n + n_1) + \langle c_{lh} \rangle (p_{lh} + p_{lh,1}) + \langle c_{hh} \rangle (p_{hh} + p_{hh,1})}, \tag{30}$$

where n_0, $p_{lh,0}$ and $p_{hh,0}$ are the equilibrium densities and the n_1, $p_{lh,1}$ and $p_{hh,1}$ have the usual meaning. If only one kind of holes is present ($p_{lh} = p_{hh} = p$), the well known formula given first by Shockley and Read results from (30):

$$R(x) = \frac{np - n_i^2}{(n + n_i)\hat{\tau}_{p0}(x) + (p + p_1)\hat{\tau}_{n0}(x)} \tag{31}$$

with

$$\hat{\tau}_{i0}(x) = \frac{1}{N_t \langle c_i \rangle}. \tag{32}$$

For constant trap distributions, N_t, the position dependence of the SRH lifetimes $\hat{\tau}_{i0}(x)$ originates from that of the averaged capture rates $\langle c_i \rangle$. The latter increases with rising electric field lowering the lifetimes in space-charge regions. In more detail,

$$\hat{\tau}_{n0}(x) = \tau_{n,0} \frac{N_c^0}{N_t} \left[\sum_{l>0} \rho_c(l) \frac{(l-S)^2}{S} \times L(l) \exp\left(\frac{E_t - l\hbar\omega_0}{kT} \right) \right]^{-1},$$
$$\hat{\tau}_{p0}(x) = \tau_{p,0} \frac{N_v^0}{N_t} \left[\sum_{l>0} \rho_v(l) \frac{(l+S)^2}{S} \times L(l) \exp\left(\frac{E_g - E_t - l\hbar\omega_0}{kT} \right) \right]^{-1}. \tag{33}$$

Within the capture rates the only variable factors are the densities of states $\rho_i(l)$ defined in Eq. (27). Therefore, in the Boltzmann case the spatially variable SRH lifetimes directly reflect the profile of the inverse tunneling probabilities in a given structure. Outside the depletion region the SRH lifetimes are indeed constants, as long as N_t is uniform.

A. Low and strong field limits

Outside the depletion region, where the potential drop is negligible, both of the normalized densities of states of light and heavy holes tend to the same zero field limit

$$\rho_{lh}^{F\to 0}(l) = \rho_{hh}^{F\to 0}(l) \to \frac{2}{\sqrt{\pi}} \frac{E_t}{(kT)^{3/2}} \sqrt{l\hbar\omega_0 - E_g + E_t} \times \Theta(l\hbar\omega_0 - E_g + E_t), \tag{34}$$

since only transitions near the band edge ($E' \simeq E_c + kT$) contribute to the SRH rate in this case. Therefore,

$$\tau_{lh,0} \langle c_{lh} \rangle p_{lh} = \tau_{hh,0} \langle c_{hh} \rangle p_{hh} = \sum_{l>0} \frac{(l+S)^2}{S} L(l) \rho_v(l) [1 - f_v(l)] \tag{35}$$

and the expression (30) reduces to

$$R(x)=N_t(x)[np_{hh}-n_0p_{hh,0}]\Big/\left(\frac{1}{\alpha\langle c_{hh}\rangle}[n+n_1]+\frac{1}{\langle c_n\rangle}[p_{hh}+p_{hh,1}]\right) \tag{36}$$

with

$$\alpha=1+\tau_{hh,0}/\tau_{lh,0}.$$

If the electric field is sufficiently strong ($\hbar\Theta_i>\hbar\omega_0$), then the most probable transition path uses deep tail states in the gap. Because of the large difference in the tunneling mass between heavy and light holes the strong field densities of states for those energies have the relation $\rho_{lh}(l)\gg\rho_{hh}(l)$, and consequently

$$\langle c_{lh}\rangle p_{lh}\gg\langle c_{hh}\rangle p_{hh} \tag{37}$$

can be assumed. Thus, only light holes participate in the recombination:

$$R(x)=N_t(x)[np_{lh}-n_0p_{lh,0}]\Big/\left(\frac{1}{\langle c_{lh}\rangle}[n+n_1]+\frac{1}{\langle c_n\rangle}[p_{lh}+p_{lh,1}]\right). \tag{38}$$

B. Trap tunneling current density in the Boltzmann case

Once the SRH rate $R(x)$ is known, the one-dimensional (1D) SRH current density can be calculated by

$$j_{SRH}=e\int dxR(x). \tag{39}$$

This current fully involves the trap tunneling contribution. The definition of the trap tunneling current density is somewhat arbitrary. We include all the phonon-assisted tunneling transitions by subtracting a "thermal background" j^{th}_{SRH} from the total SRH current:

$$j^{tun}_{SRH}=j_{SRH}-j^{th}_{SRH}. \tag{40}$$

j^{th}_{SRH} is caused by transitions with a recombination energy greater or equal than the gap energy. Therefore, j^{tun}_{SRH} stems from regions with noticeable electric field only.

Due to (39) the corresponding tunneling rate $R^{tun}(x)=R(x)-R^{th}(x)$ has to be known. In the case of Boltzmann statistics a most simple result follows in a two-band model ($m_{lh}=m_{hh}=m_v$):

$$R^{tun}=R^{th}\left\{\left[\frac{1}{\langle c_p^{th}\rangle}(n+n_1)+\frac{1}{\langle c_n^{th}\rangle}(p+p_1)\right]\Big/\left[\frac{1}{\langle c_p\rangle}\times(n+n_1)+\frac{1}{\langle c_n\rangle}(p+p_1)\right]-1\right\}. \tag{41}$$

This expression can be further simplified under the following assumptions: (a) $m_c=m_v$, (b) $E_t=E_g/2$, (c) $l^*\gg S$. It is clear from the definition of $\langle c_i\rangle$, that these restrictions bring about the relation

$$\frac{\langle c_p\rangle}{\langle c_p^{th}\rangle}=\frac{\langle c_n\rangle}{\langle c_n^{th}\rangle}=\frac{\langle c\rangle}{\langle c_{th}\rangle}, \tag{42}$$

and the trap tunneling rate becomes

$$R^{tun}=R^{th}\left(\frac{\langle c\rangle}{\langle c^{th}\rangle}-1\right). \tag{43}$$

The term in brackets is a sum over thermally weighted tunneling factors

$$\left(\frac{\langle c\rangle}{\langle c^{th}\rangle}-1\right)=Z^{-1}\sum_{l>0}l^2I_l\exp\left(-\frac{l\hbar\omega_0}{2kT}\right)\times[\rho(l)-\rho^{F=0}(l)] \tag{44}$$

with

$$Z=\sum_{l>l_0+1}l^2I_l\exp\left(-\frac{l\hbar\omega_0}{2kT}\right)\rho^{F=0}(l).$$

The main contribution to the tunneling current comes from a fraction of the depletion region, where the field strength has a maximum. There, l^* can be supposed to be much smaller than $l_0=\text{int}(E_t/\hbar\omega_0)$. This enables to replace the electro-optical function $\mathscr{F}(y)$ by its asymptotic form (WKB approximation), and the tunneling factor becomes

$$[\rho(l)-\rho^{F=0}(l)]\to\rho(l)\simeq\frac{1}{4\sqrt{\pi}}\frac{E_t}{kT}\frac{1}{k_{th}\xi(l)}\times\exp[-2S_{E_l}(x)]. \tag{45}$$

The approximations made so far in this section allow for the tunneling current density in an analytical form, if integrable potential shapes are used. As a simple example we consider a surface potential (constant donor concentration N_D, depletion approximation). The value of the electrostatic potential $\varphi(x)$ at the boundary of the space-charge region x_n is denoted by φ_n, its value at the surface (x_s) by φ_s. The band edges in the depletion region then are given by

$$E_{c,v}(x)=E^0_{c,v}-e\varphi_n+eU+\frac{kT}{2}\frac{(x-x_n)^2}{L_D^2}. \tag{46}$$

The point x_n depends on external voltage U:

$$x_n=x_s-L_D\sqrt{2(U_{bi}-U)/U_T}. \tag{47}$$

In (46), (47) L_D denotes the Debye length $L_D=(\epsilon U_T/4\pi eN_D)^{1/2}$, U_{bi} is the built-in potential $U_{bi}=\varphi_n-\varphi_s$, and U_T the thermal voltage. After some algebra the action integral can be expressed exactly in form of a power row:

$$S_{E_l}(x)=\sqrt{2}L_Dk_{th}\frac{E_t-l\hbar\omega_0}{kT}\sum_{m=1}^{\infty}\frac{1}{4m^2-1}\times\left(\frac{\sqrt{2}L_D\sqrt{(E_t-l\hbar\omega_0)/kT}}{x-x_n}\right)^{2m-1}. \tag{48}$$

The convergence of the row (48) is guaranteed by the fact, that for a given phonon number l the first x value fulfills the relation

$$x \gg x_n + \sqrt{2} L_D \sqrt{(E_t - l\hbar\omega_0)/kT}. \quad (49)$$

It was mentioned above that only a small region in the vicinity of the surface position x_s contributes to the tunneling current. Inserting x_s for x in (48) and using (47) shows that one can neglect all terms with $m>1$ in the row (48) if

$$E_t - l\hbar\omega_0 \ll eU_{\rm bi} - eU \quad (50)$$

holds. Since the left-hand side is of the order $E_g/4$ (WKB approximation), this restricts the external voltage to the reverse bias branch here.

Then, the result for the action is

$$S_{E_l}(x) = \frac{2}{3}\frac{L_D^2 k_{\rm th}}{x - x_n}\left(\frac{(E_t - l\hbar\omega_0)}{kT}\right)^{3/2}. \quad (51)$$

The quantity $\xi(l)$ in the denominator of the tunneling factor $\rho(l)$ (45) can be calculated in the same way. One obtains

$$\xi(l) = \frac{1}{\sqrt{2}}\frac{L_D^3}{(x - x_n)^2}\left(\frac{(E_t - l\hbar\omega_0)}{kT}\right)^{3/2}. \quad (52)$$

With (43), (44), (45), (51), and (52) the trap tunneling rate can be written in the form

$$R^{\rm tun}(x) = \frac{\langle R^{\rm th}\rangle (x - x_n)^2 \sqrt{kT/E_t}}{\sqrt{8\pi} Z L_D^3 k_{\rm th}} \times \sum_{\substack{l>0 \\ (l \ll l_0)}} \frac{l^2 I_l}{(1 - l/l_0)^{3/2}} \exp\left[-\frac{l\hbar\omega_0}{2kT} - \frac{4}{3}\frac{L_D^2 k_{\rm th}}{x - x_n}\left(\frac{E_t - l\hbar\omega_0}{kT}\right)^{3/2}\right]. \quad (53)$$

In (53) the thermal rate $R^{\rm th}$ was replaced by an averaged value

$$\langle R^{\rm th}\rangle = \frac{N_t(e^{U/U_T} - 1)}{\tau_{\rm eff}(T)}.$$

Now, we can integrate (53) approximately to get the trap tunneling current density. The result is

$$j_{\rm SRH}^{\rm tun}(U) = j_0(U) \sum_{\substack{l>0 \\ (l \ll l_0)}} \frac{l^2 I_l}{(1 - l/l_0)^3} \exp\left[-\frac{l\hbar\omega_0}{2kT} - \frac{4}{3}\frac{L_D k_{\rm th}}{\sqrt{2}}\left(\frac{E_t - l\hbar\omega_0}{kT}\right)^{3/2} \frac{U_T}{\sqrt{U_{\rm bi} - U}}\right] \quad (54)$$

with

$$j_0(U) = \frac{3}{2}\frac{e\langle R^{\rm th}\rangle}{\sqrt{2\pi} Z L_D k_{\rm th}^2}\frac{(eU_{\rm bi} - eU)^2}{E_t^2}.$$

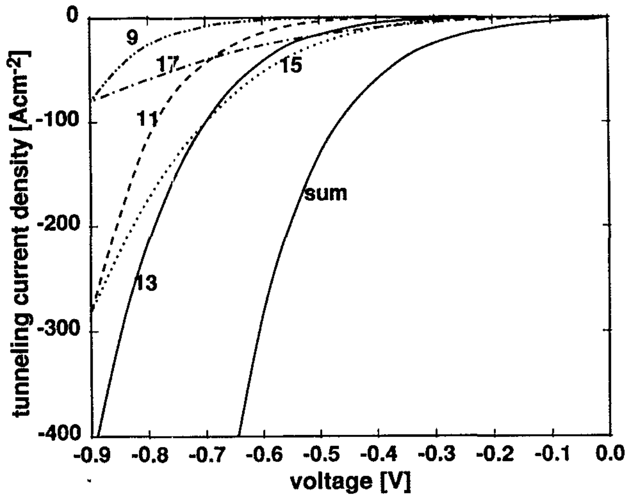

FIG. 4. Reverse bias tunneling characteristics of various n-phonon transitions (n indicated by the numbers) and the resulting sum. Parameters: $T=333$ K, $S=3.5$, $\hbar\omega_0 = 33$ meV, $\tau_{\rm eff} = 10^{-6}$ s, $E_g = 1.5$ eV, $E_t = 0.75$ eV, $m_c = 0.066m_0$, $N_t = 10^{14}$ cm^{-3}, $\epsilon_s = 12.9$, $N_D = 10^{17}$ cm^{-3}, $U_{\rm bi} = 0.45$ eV.

Formula (54) represents a superposition of a series of tunneling characteristics, valid only in the reverse bias branch here, each produced by band-to-trap transitions of electron-hole pairs with assistance of $2\times l$ phonons. The number of phonons explicitly occurs in the WKB tunneling exponent, lowering the energy barrier E_t and, consequently, enhancing the tunneling probability. The latter is multiplied with a thermal weight which decreases exponentially with rising l. Both tendencies result in an optimum transition assisted by $2\times l^*$ phonons ($l^* \ll l_0$ has been assumed in this section). In previous papers (e.g., Ref. 8) only the zero phonon part ($l=0$) was taken for the trap tunneling current, and a thermal SRH current was added neglecting all the phonon-assisted tunneling transitions. At temperatures for which the SRH recombination is important, the contribution of the zero phonon transition is negligible. This is clearly demonstrated in Fig. 4, where Eq. (54) was analyzed for various phonon numbers. GaAs/EL2 parameters were used in the described surface potential model. The optimum phonon number is $l^*=13$, while the half-gap needs 22 phonons for the carrier to be transported directly on a purely thermal way. The total tunneling characteristic is mainly the result of transitions with shares of 9 to 17 phonons.

VII. SRH CURRENT-VOLTAGE CHARACTERISTICS

A. Uniform trap distribution

The following results are based on the general expression (25) for the recombination rate $R(x)$ and the above described artificial, but nevertheless instructive, partition into a tunneling part $R^{\rm tun}(x)$ and a thermal part $R^{\rm th}(x)$. For application we consider a n^+p junction in $Hg_{0.8}Cd_{0.2}Te$ (at $x=0.5\ \mu$m). In such a structure heavy and light holes are involved and degeneracy effects can be expected at least in forward bias operation. The trap con-

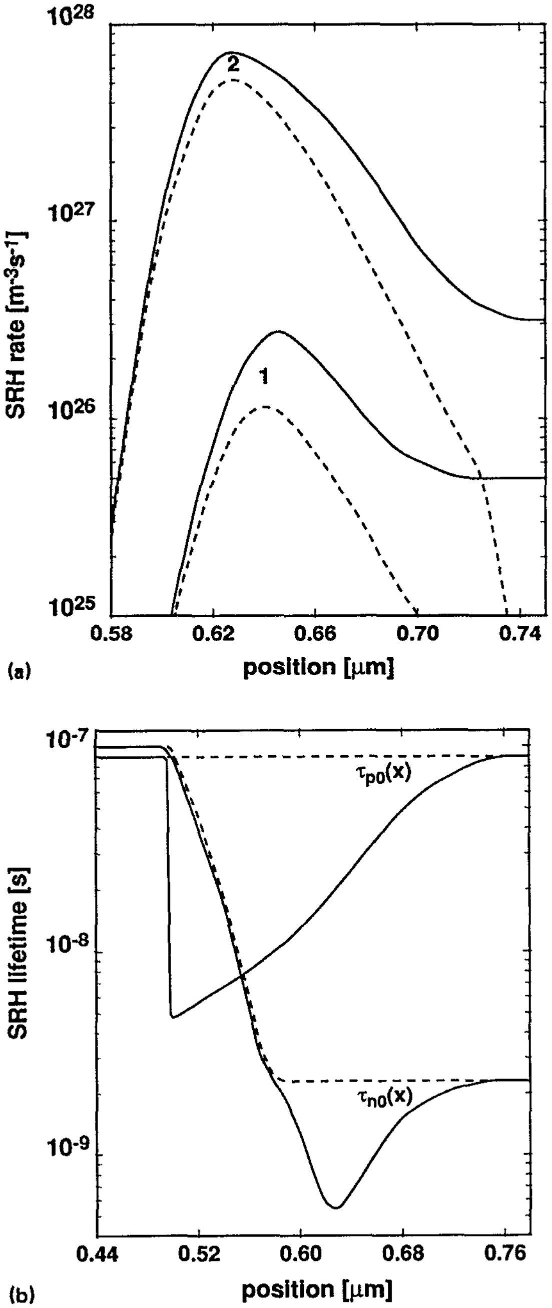

FIG. 5. (a) SRH rate (solid curve) and trap tunneling rate (dashed curve) in a $Hg_{0.8}Cd_{0.2}Te$ n^+p junction for two biases: (1) $U = +0.001$ V, (2) $U = -0.05$ V. Parameters: $x_{pn} = 0.5$ μm, $T = 77$ K, $S = 4.4$, $\hbar\omega_0 = 6$ meV, $N_t = 10^{18}$ cm^{-3}, $\tau_{n,0} = \tau_{\text{lh},0} = 10^{-6}$ s, $\tau_{\text{hh},0} = 10^{-4}$ s, $E_t = 42.3$ meV, $E_g = 84.7$ meV, $n^+ = 10^{18}$ cm^{-3}, $N_A = 10^{16}$ cm^{-3}. (b) SRH lifetimes $\hat{\tau}_{i0}(x) = (N_c\langle c_i\rangle)^{-1}$ in the $Hg_{0.8}Cd_{0.2}Te$ n^+p junction of (a) for -50 mV bias (solid curves). The dashed lines indicate the shape of the SRH lifetimes, if only phonon-assisted trap tunneling transitions are taken into account. For parameters see (a).

centration $N_t(x)$ is assumed to be constant. Then, the spatial variability of the rate $R(x)$ is due to the occupancies and the field dependent densities of states only. A plot of $R(x)$ and $R^{\text{tun}}(x)$ for different biases is shown in Fig. 5(a). The maxima occur in the p region of the space-charge layer as a result of decreasing field strength and a rising number of free electron states. There, the relative contribution of tunneling transitions is more than 50%. This contribution enhances in the direction of the increasing field.

It is useful to study the field effect on the SRH lifetimes for this example. Therefore, we write the general expression (25) for the total rate $R(x)$ in the form

$$R(x) = np_{\text{hh}}\left[1 - \exp\left(-\frac{F_n - F_p}{kT}\right)\right] \Big/ \left(\frac{n\hat{\tau}_{p0}(x)}{f_t^n} + \frac{p_{\text{hh}}\hat{\tau}_{n0}(x)}{(1 - f_t^p)}\right). \quad (55)$$

The Boltzmann limit follows from $f_t^n \to n/(n + n_1)$ and $1 - f_t^p \to p_{\text{hh}}/(p_{\text{hh}} + p_{\text{hh},1})$. The lifetime $\hat{\tau}_{n0}(x)$ is given by (32), $\hat{\tau}_{p0}(x)$ by

$$\hat{\tau}_{p0}^{-1}(x) = N_t\left(\frac{p_{\text{lh}}}{p_{\text{hh}}}\langle c_{\text{lh}}\rangle + \langle c_{\text{hh}}\rangle\right). \quad (56)$$

These lifetimes are plotted in Fig. 5(b) as function of x using the parameters of Fig. 5(a). In the case of the nondegenerated holes, the lifetime has the same constant value on both sides outside the space-charge layer. Within the depletion region it reflects the electric field distribution of the asymmetrical junction. Without taking into account the trap tunneling transitions, $\hat{\tau}_{p0}(x)$ would be constant. This does not apply to the electron lifetime $\hat{\tau}_{n0}(x)$, which decreases from the n^+ to the p region by nearly two orders of magnitude due to the transition from degeneracy to nondegeneracy. The field effect on $\hat{\tau}_{n0}(x)$ via the density of states is quenched with starting degeneracy, since there are no empty states left for tunneling processes.

A SRH current-voltage characteristic of this example is shown in Fig. 6. The integration of $R(x)$ was broken off at 0.8 μm (device length). Then the reverse bias branch is completely determined by phonon-assisted trap tunneling, whereas in the forward branch the well known tunnel "bump" emerges for $T \leqslant 130$ K. Since we have assumed a continuous trap distribution N_t, the SRH current increases continuously with rising voltage.

B. Discrete trap configurations

First we consider a delta-like trap density

$$N_t = \nu_t\delta(x - x_0). \quad (57)$$

In this simple model there is only one phonon ladder at the point x_0, which is pinned to the trap energy level. Consequently, the series of phonon sublevels moves together with the band edges, if an external voltage is applied.

In principle, phonon substructures in the forward $I-U$ characteristic can be expected now by two reasons. Firstly, the quasi-Fermi levels move on the energy scale relative to the phonon ladder switching on or off a new multiphonon transition channel, respectively, according to their velocity ratio. Since the Fermi edge is smeared out by kT, the fluctuations cannot be sharp and vanish for $kT \simeq \hbar\omega_0$.

Second, due to the lowering of the potential barrier, sublevel after sublevel will be excluded from tunneling,

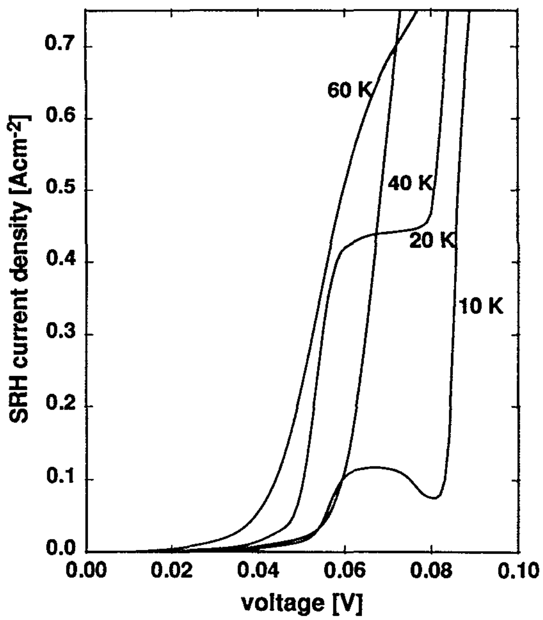

FIG. 6. SRH current density of the $Hg_{0.8}Cd_{0.2}Te$ n^+p diode of Fig. 5(a) for different temperatures: (1) $T=10$ K, (2) $T=20$ K, (3) $T=40$ K, (4) $T=60$ K. An acceptor binding energy of 6 meV and Fermi level pinning at the conduction-band edge in the n region have been assumed. For parameters see Fig. 5(a).

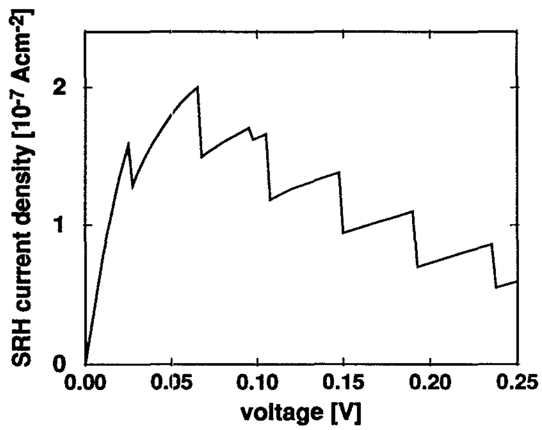

FIG. 7. SRH forward current density of a δ-like trap configuration in the GaAs surface potential example of Fig. 4. The trap distance from the surface is 2 nm. Parameters: $T=300$ K, $\tau_{n,0} = 10^{-15}$ s, $\tau_{p,0} = 0$, $U_{bi} = 0.85$ V, for other parameters see Fig. 4.

since band states at the same energy (density of states limitation) do not remain. At these voltages a dip in the current curve appears. Looking at the surface potential example of Sec. VI B one can easily calculate the dip positions. With (46) and (47) the dip resulting from the loss of the n phonon process for electrons appears at the voltage

$$U_n = U_{bi} - \left(\sqrt{\frac{(E_t - n\hbar\omega_0)}{e}} + \sqrt{\frac{U_T}{2}}\frac{x_s - x_0}{L_D}\right)^2. \tag{58}$$

If the trap is located near the surface ($x_s - x_0 < L_D$), the dip spacing is given by the phonon energy: $\Delta U_n \simeq \hbar\omega_0/e$.

Figure 7 shows the sawtooth-like characteristic of this case. A doping level of $N_D = 5 \times 10^{18}$ cm^{-3} and a trap distance $x_s - x_0 = 2$ nm have been used. The hole lifetime was set to zero in order to demonstrate the dips from the conduction band to deep level processes separately. The lifetime constant τ_{n0} and the trap concentration ν_t have been chosen such that the field-independent SRH lifetime is of the order 1 ns.

The dips resulting from the loss of the n-phonon process for holes appear at the voltages

$$U_n = U_{bi} - \frac{L_D^2}{(x_s - x_0)^2}\frac{U_T}{2}\left(\frac{E_g - E_t - n\hbar\omega_0}{kT} + \frac{(x_s - x_0)^2}{2L_D^2}\right)^2. \tag{59}$$

In this case near the surface, approximately

$$U_n \simeq U_{bi} - \frac{L_D^2(\hbar\omega_0)^2}{2(x_s - x_0)^2}\frac{1}{ekT}(l_{0,p} - n)^2 \tag{60}$$

with $l_{0,p} = \mathrm{int}[(E_g - E_t)/\hbar\omega_0]$. Here the dip spacing is larger than in the other case, and it decreases with rising voltage. Due to the assumed square potential the dip positions are distributed quadratically, and the dip spacing is reduced by the factor $2(l_{0,p} - n) - 1$ on the voltage axis.

In a real semiconductor the traps are distributed on the crystal grid:

$$N_t(x) = \nu_t(x) \sum_i \delta(x - x_i) \tag{61}$$

with $\Delta x = x_{i+1} - x_i$ being a distance of the order of the lattice constant. In this model the above described fluctuations (58) and (59) will superimpose and give rise to a strongly structured forward $I_{SRH} - U$ characteristic. Figure 8 shows two theoretical examples of a GaAs surface potential with $U_{bi} = 0.85$ V and donor concentrations of 1×10^{17} cm^{-3} and 5×10^{17} cm^{-3}, respectively. In the higher doped device a pronounced tunnel bump with narrow multiphonon structures is observed. With decreasing doping level the recombination region extends and the SRH current rises. At the same time the dip spacing increases ($L_D^2 \sim N_D^{-1}$) and the bump becomes less marked.

VIII. SUMMARY AND CONCLUSIONS

It was the goal of this paper to uniform the description of thermal SRH recombination and trap-assisted tunneling in semiconductor space-charge regions. Therefore, the thermal capture rates which enter the SRH lifetimes were defined in a multiphonon picture with field-dependent densities of band states. Inhomogeneous fields could be included using an approximate envelope for these states. In the most frequent case of Boltzmann statistics the spatial variability of the SRH lifetimes reflects the field distribution in the device, if the trap density N_t is constant.

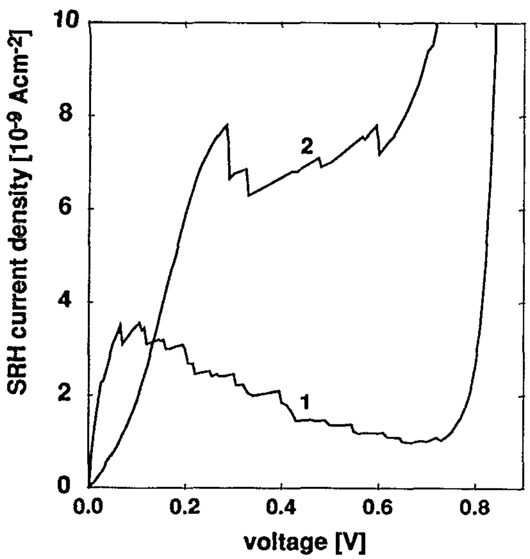

FIG. 8. SRH forward current density of the GaAs surface potential of Fig. 7 with uniformly distributed traps on a grid with (1) $\Delta x_i = d = 0.565$ nm, $N_D = 5 \times 10^{17}$ cm^{-3} and (2) $\Delta x_i = 2d$, $N_D = 1 \times 10^{17}$ cm^{-3}. Parameters: $\tau_{n,0} = \tau_{p,0} = 10^{-15}$ s, other parameters unchanged.

Instead of subdividing the recombination processes into two classes: tunneling (horizontal) and thermal (vertical), in the presented model every single electron-hole recombination is a combination of multiphonon capture (emission) and tunneling into (from) a multiphonon sublevel. The probability of those processes can be larger by several orders of magnitude.

The model was developed for Fermi statistics and three bands in order to take account of the injection case in highly doped small gap systems. An analytical expression for the trap tunneling current could be derived under certain assumptions (WKB limit, reverse bias branch, midgap level, equal masses). The total breakdown characteristic was shown to be superimposed of individual n-phonon tunnel branches.

The recombination model was applied to a $Hg_{0.8}Cd_{0.2}Te$ n^+p junction and a GaAs surface potential. Besides the trap energy level E_t and two multiphonon parameters (S, $\hbar\omega_0$) a lifetime constant $\tau_{i,0}$ for each kind of carriers has to be given, which includes the transition matrix element. E_t, S, and $\hbar\omega_0$ are available from deep level transient spectroscopy (DLTS) (DDLTS), whereas the $\tau_{i,0}$ have to serve as fit parameters.

As part of the SRH rate the nonlocal rate of deep level to band tunnel transitions enters the Van Roosbroeck system of semiconductor device equations[27] and determines the total current density together with the remaining recombination processes. A computer routine has been developed for use in existing device simulators. Here the problem had to be solved to find the classical turning points of the electrostatic potential, which is calculated self-consistently on a certain finite-element grid,[28] for every multiphonon energy sublevel at a given grid point.

As long as the trap concentration N_t is considered continuous it only scales the SRH lifetimes. Any discrete trap distribution yields sharp fluctuations of the forward $I_{SRH} - U$ characteristic. Such oscillations result from the fact that each n-phonon tunneling transition becomes density of states limited if the external voltage is increased. On the other hand, the separation of the sublevels has its origin in the supposed sharpness of the one mode multiphonon spectrum (Einstein model). Various damping mechanisms, like phonon dispersion, phonon-phonon collisions, etc., smooth out the lineshape function and allow the observation of the described fluctuations, if at all, at very low temperatures.

Sharply structured forward I-U characteristics, similar to that described in this paper, have been observed recently[29] at GaAs Schottky mixer diodes under strong illumination with an infrared laser beam at room temperature. There, a tunnel bump with short period dips emerged indicating multiphoton-assisted trap tunneling.

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